Computational Complexity

CPSC 468/568, Spring 2016
Time: Tu & Th, 2:30-3:45 pm
Room: AKW 000

http://zoo.cs.yale.edu/classes/cs468/

Partial Topic Outline

- Complexity classes (P, NP, L, NL, *etc.*)
- Reductions and completeness
- The roles of, *e.g.*,
  - Randomness
  - Interaction
  - Approximation

? Communication complexity

Requirements (undergrads)

- Modest reading assignments, mostly in Arora and Barak, **Computational Complexity: A Modern Approach**, Cambridge Univ. Press.
- 6 Written HW Assignments, each worth 10% of the course grade
- 2 In-Class Exams, each worth 20% of the course grade
- No final exam during exam week

Requirements (grad students)

- Modest reading assignments, mostly in Arora and Barak, **Computational Complexity: A Modern Approach**.
- 6 Written HW Assignments, each worth 9% of the course grade
- Note taking and/or lecturing, worth 6% of the course grade
- 2 In-Class Exams, each worth 20% of the course grade
- No final exam during exam week

Tentative Schedule

Thurs, February 4: First HW Assignment Due
Thurs, February 18: Second HW Assignment Due
Thurs, March 3: Third HW Assignment Due
Tues, March 8: First In-Class Exam
Fri, March 11: Spring Semester Drop Date
Thurs, March 31: Fourth HW Assignment Due
Tues, April 12: Fifth HW Assignment Due
Tues, April 26: Sixth HW Assignment Due
Thurs, April 28: Second In-Class Exam

Rules and Guidelines

- Deadlines are firm.
- Late penalty: 5% per day for at most 7 days, after which solutions are posted and HWs not yet turned in receive a grade of zero.
- Announcements and assignments will be posted on the class webpage (as well as conveyed in class).
- **Try to do the HW on your own. If you work in a group to solve a HW problem, identify the group members on your HW paper. If you use any sources except the textbook and classnotes, identify them.**
- Pick up your graded homeworks and exams promptly, and tell the TA promptly if one is missing.

Instructor: Joan Feigenbaum

Office: AKW 512

Office Hours: TBA and by appointment

Phone: 203-432-6432

Assistant: Judi Paige

(judi.paige@yale.edu, 203-436-1267,
AKW 507a, 8:30 am – 4:30 pm M-F)

Note: Do not send email to Professor Feigenbaum, who suffers from RSI. Contact her through Ms. Paige or the TA.

TA: Debayan Gupta

Office: AKW 503

Email: Debayan.Gupta@yale.edu

Office Hours:

Tues and Thurs, 4 – 5 pm

and by appointment

If you're undecided …

Check out:

- **http://zoo.cs.yale.edu/classes/cs468/spr15/**
- **http://www.cs.princeton.edu/theory/complexity/**
  (draft of textbook by Sanjeev Arora and Boaz Barak of Princeton)
- **http://www.cs.berkeley.edu/~luca/cs278-02/**
  (a complexity-theory course taught by Luca Trevisan at Berkeley in 2002)
- **http://fileadmin.cs.lth.se/cs/Personal/Rolf_Karlsson/bk/retro.pdf**
  ("NP-Completeness: A Retrospective," by Christos Papadimitriou, 1997 International Colloquium on Automata, Languages, and Programming)

Questions?

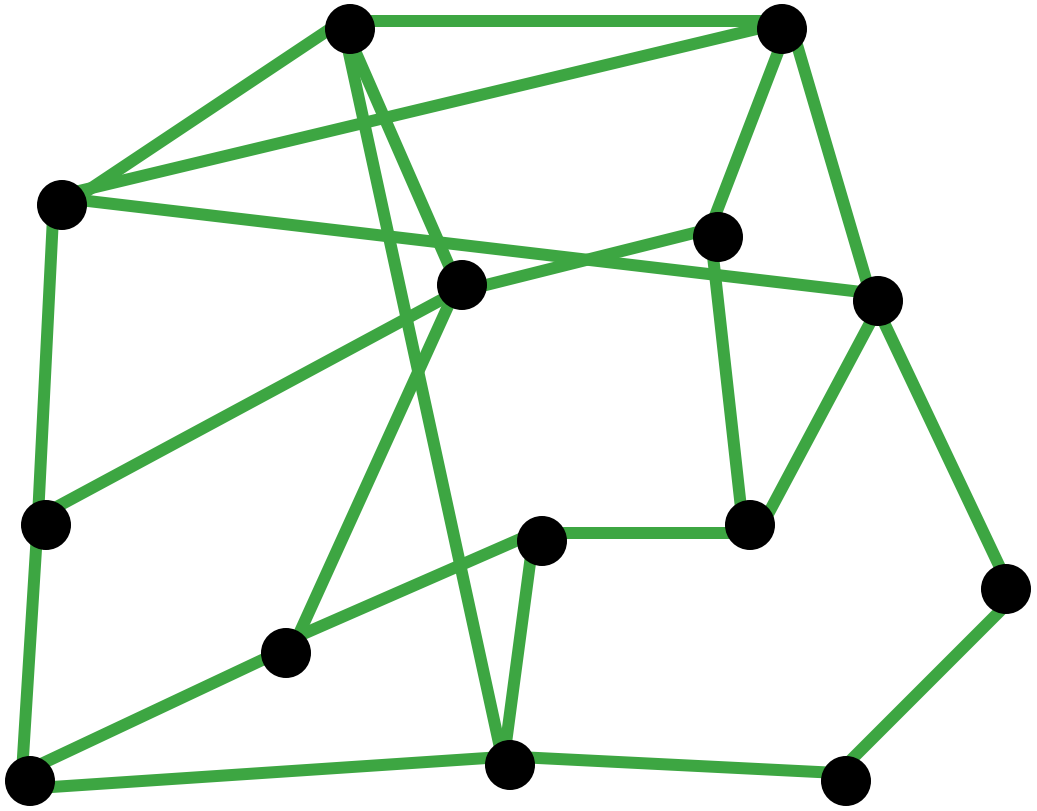
Introduction to Complexity Classes

Computational Complexity Themes

- "Easy" vs. "Hard"
- Reductions (Equivalence)
- Provability
- Randomness

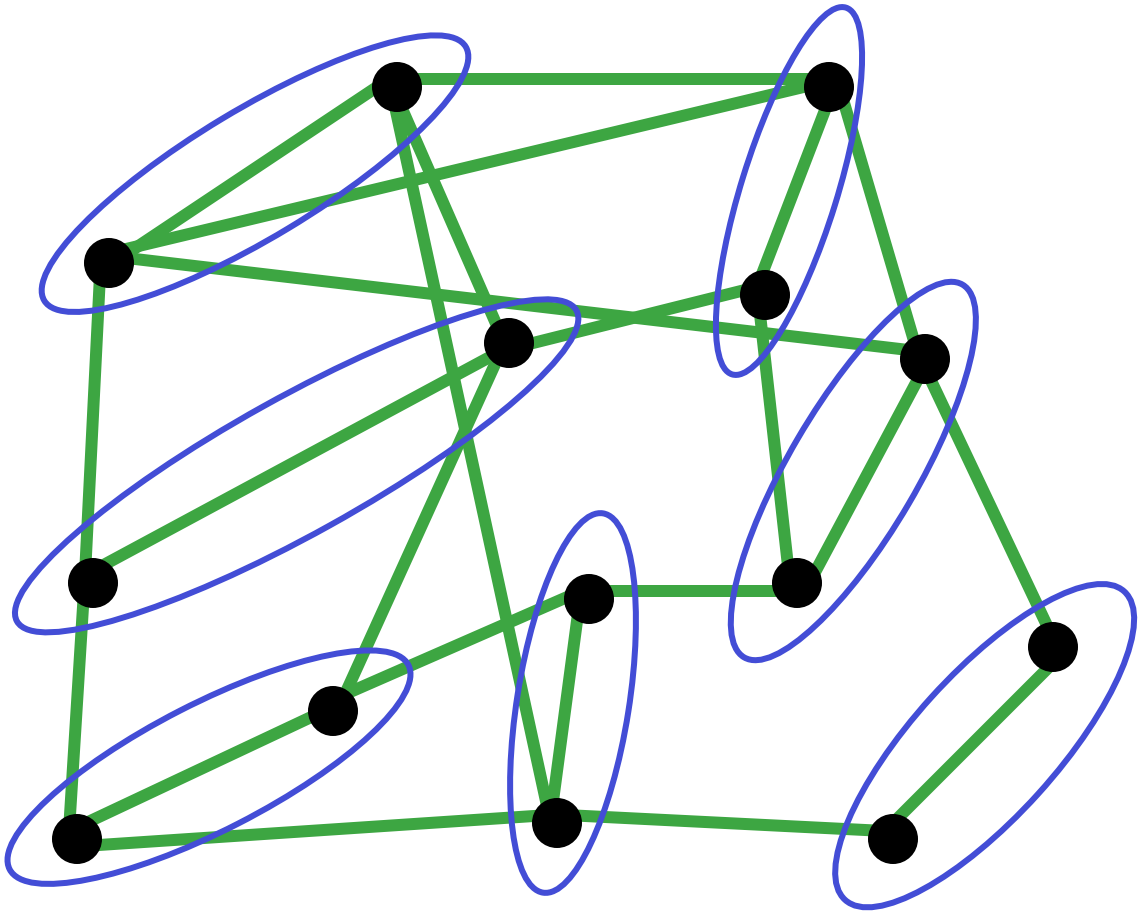
Poly-Time Solvable

Nontrivial Example: Matching



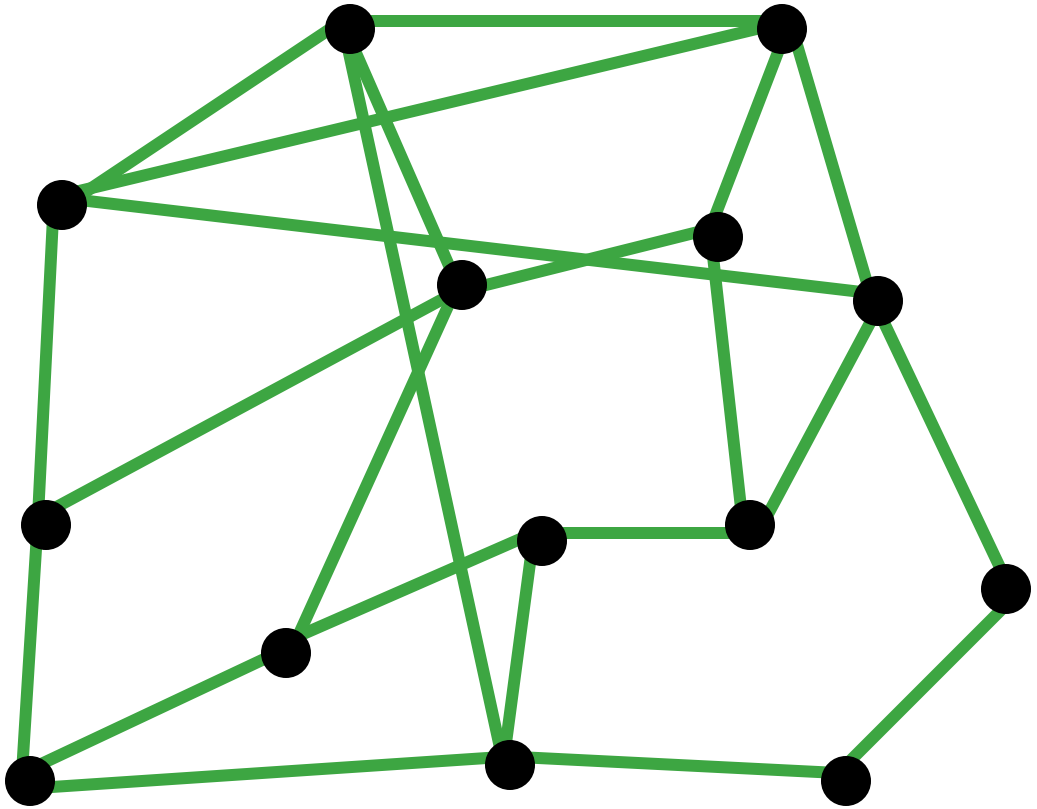
Poly-Time Solvable

Nontrivial Example: Matching



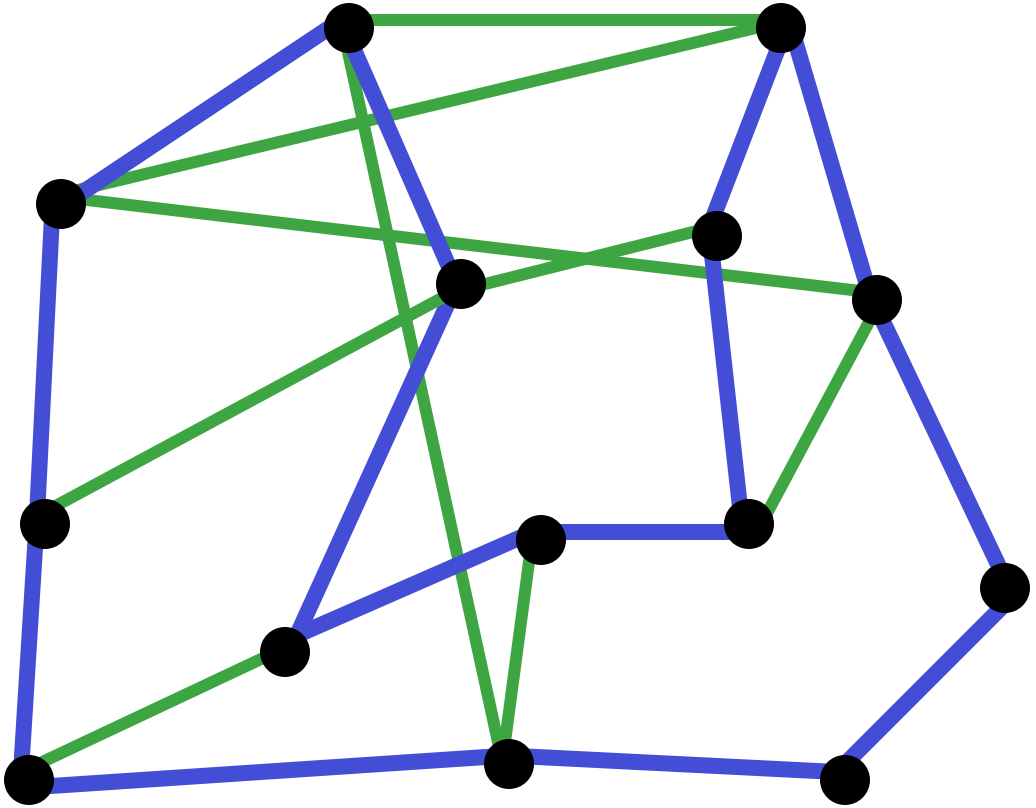
Poly-Time Verifiable

Trivial Example: Hamiltonian Cycle



Poly-Time Verifiable

Trivial Example: Hamiltonian Cycle



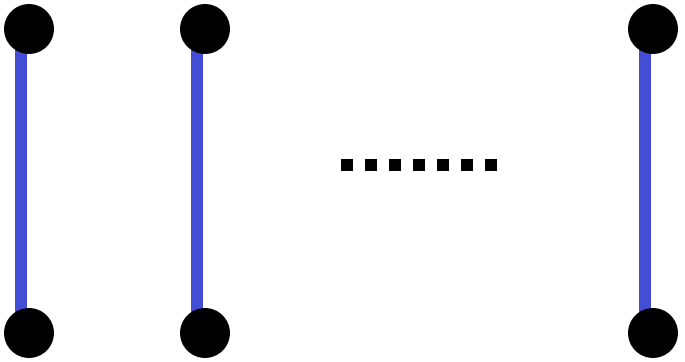
- Is it Easier to Verify a Proof than to Find one?

- Fundamental Conjecture of Computational Complexity:

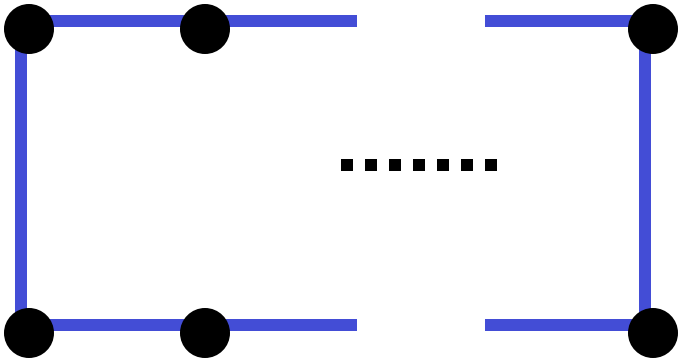
$$P \neq NP$$

Distinctions

Matching:



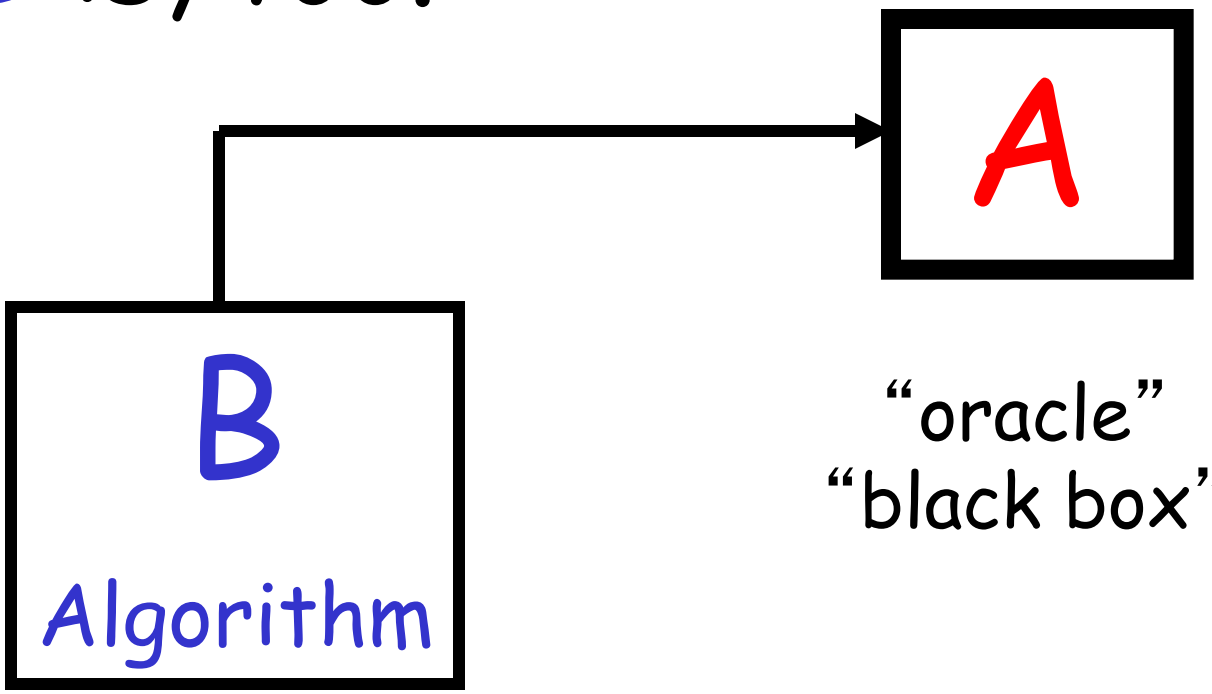
HC:



Fundamentally Different

Reduction of B to A

If A is "Easy,"
then B is, too.



- NP-completeness
- P-time reduction
- Cook's theorem

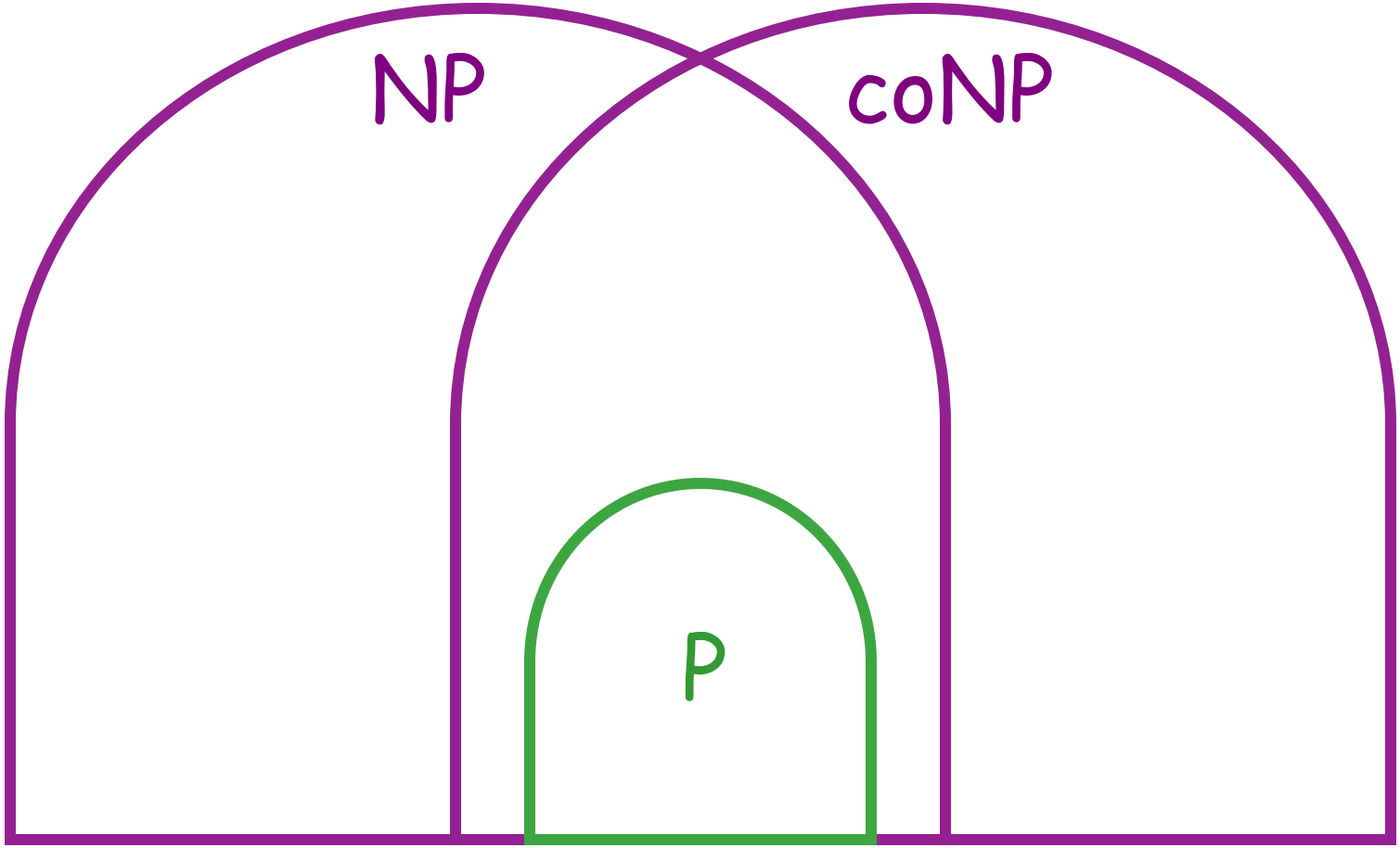
  If $B \in NP$, then

  $B \leq_{P\text{-time}} SAT$
- HC is NP-complete

Equivalence

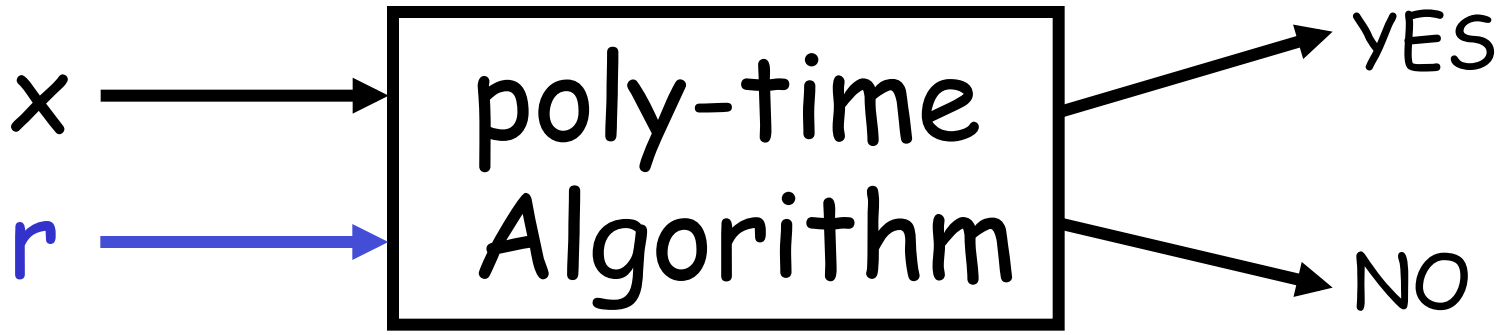
- NP-complete problems are an equivalence Class under polynomial-time reductions.
- 10k's problems
- Diverse fields

  Math, CS, Engineering, Economics, Physical Sci., Geography, Politics…


NP
coNP
P

Random poly-time Solvable

$x \in L?$



$x \in \{0,1\}^n$

$r \in \{0,1\}^{poly(n)}$

Probabilistic Classes

RP

- $x \in L \rightarrow$ "yes" w.p. $\frac{3}{4}$
- $x \notin L \rightarrow$ "no" w.p. 1

coRP

- $x \in L \rightarrow$ "yes" w.p. 1
- $x \notin L \rightarrow$ "no" w.p. $\frac{3}{4}$

(Outdated) Nontrivial Result

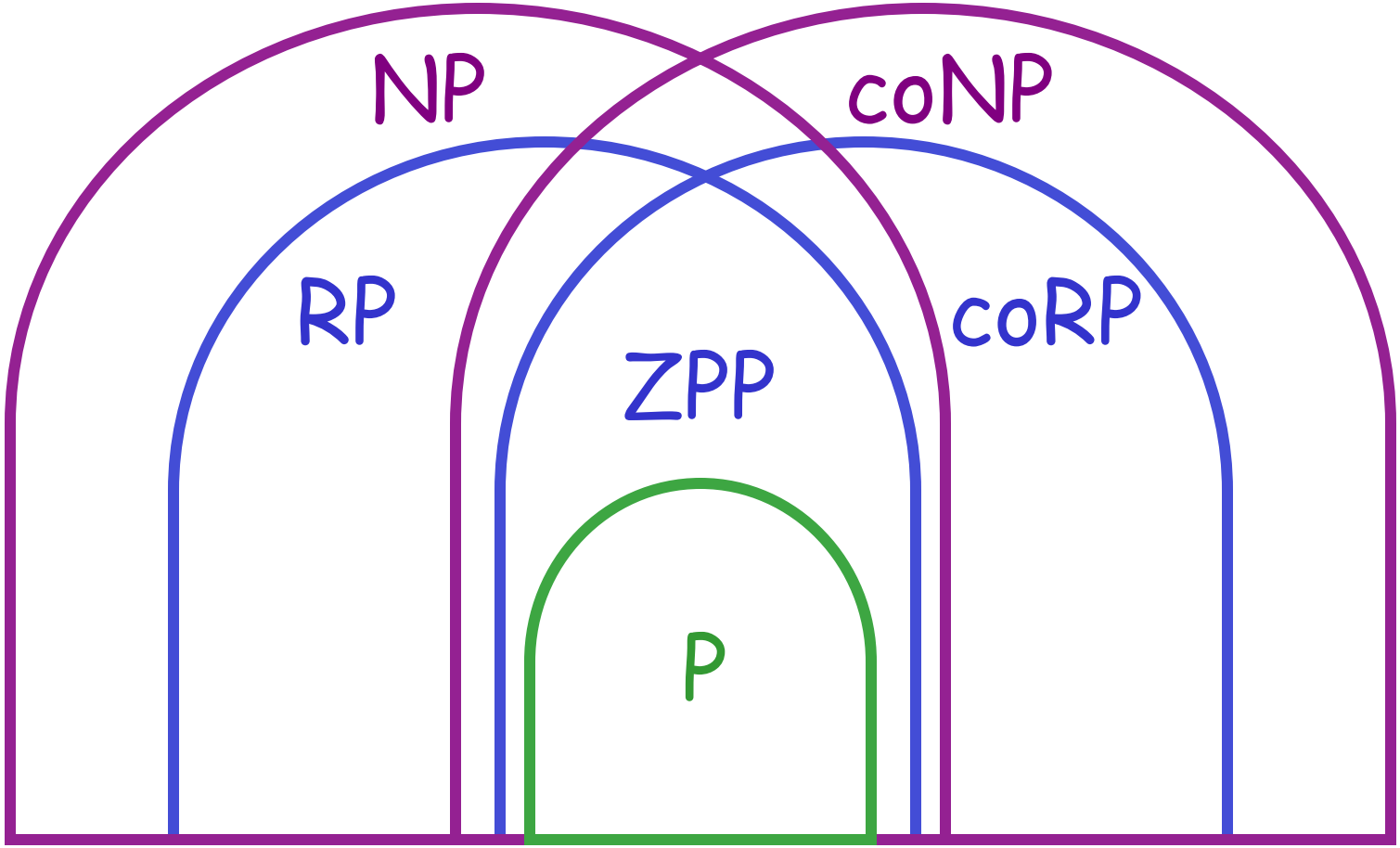
PRIMES $\in$ ZPP (= RP $\cap$ coRP)

Two-sided Error

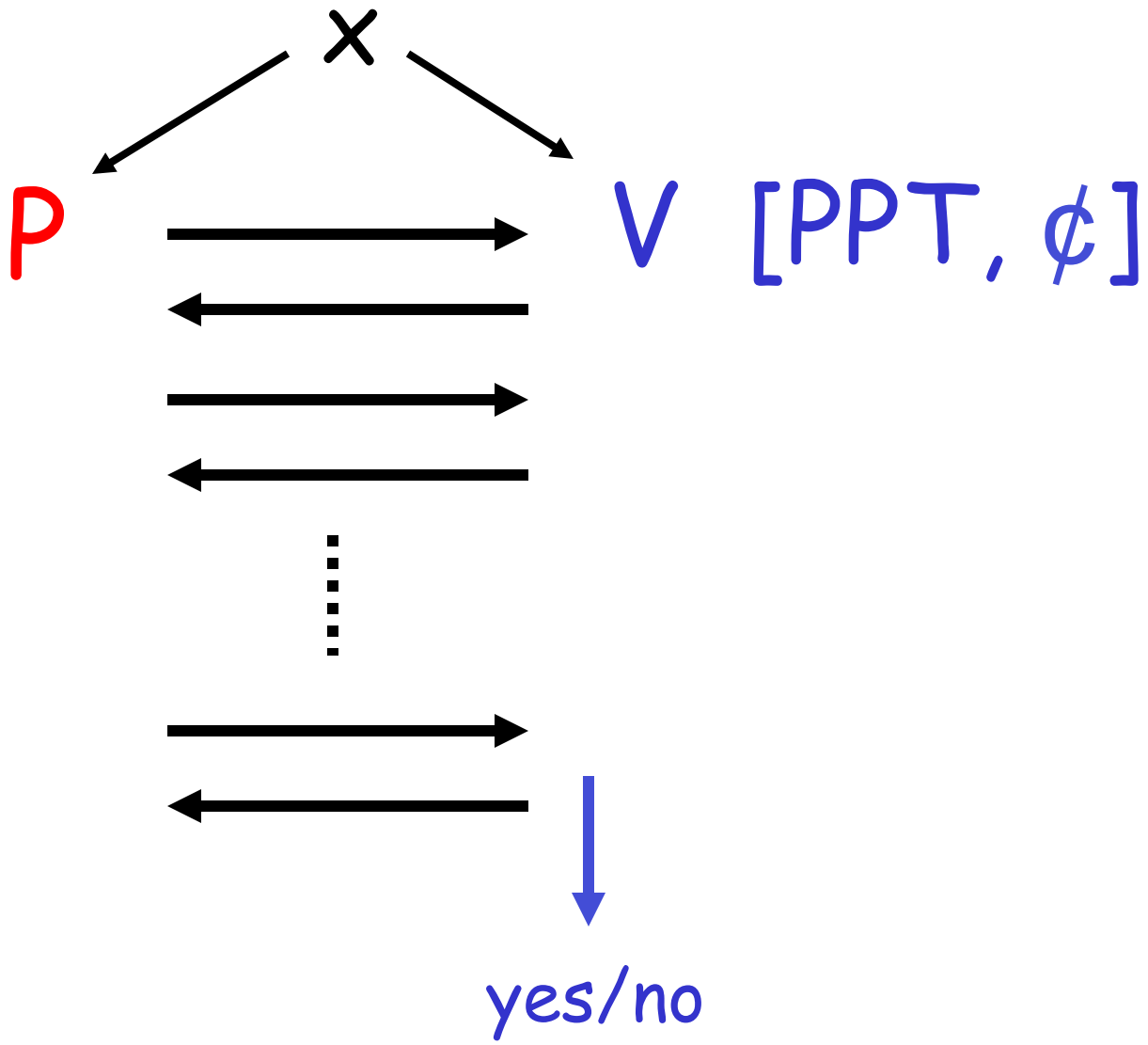
BPP $\begin{cases} x \in L \rightarrow \text{“yes”} & \text{w.p. } \frac{3}{4} \\ x \notin L \rightarrow \text{“no”} & \text{w.p. } \frac{3}{4} \end{cases}$

Question to Audience: Is there a BPP set not known to be in RP or coRP?

Note: QR is in NP and coNP but not known to be in RP or coRP.


NP
coNP
RP
coRP
ZPP
P

Interactive Provability



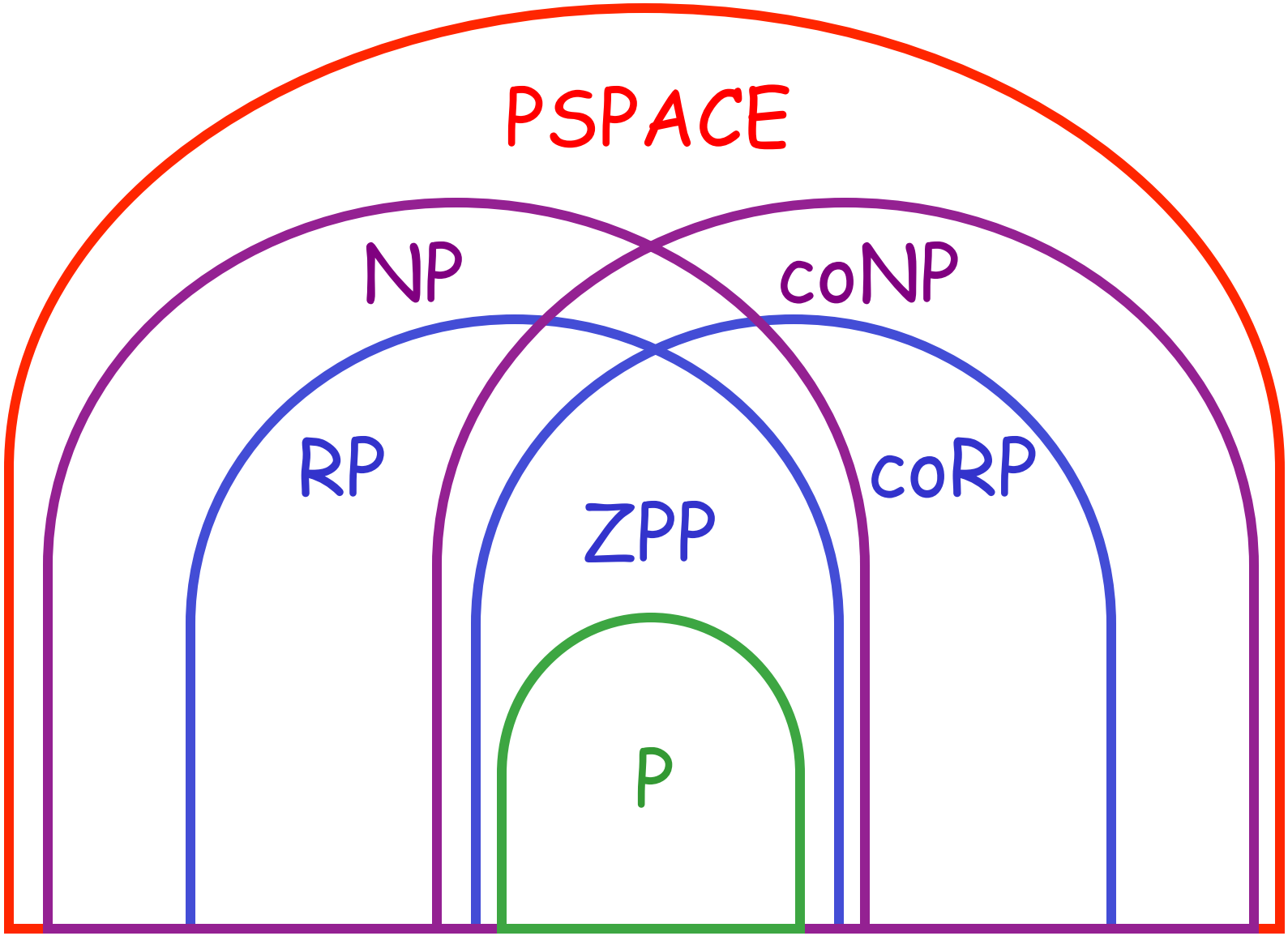
$L \in IP$

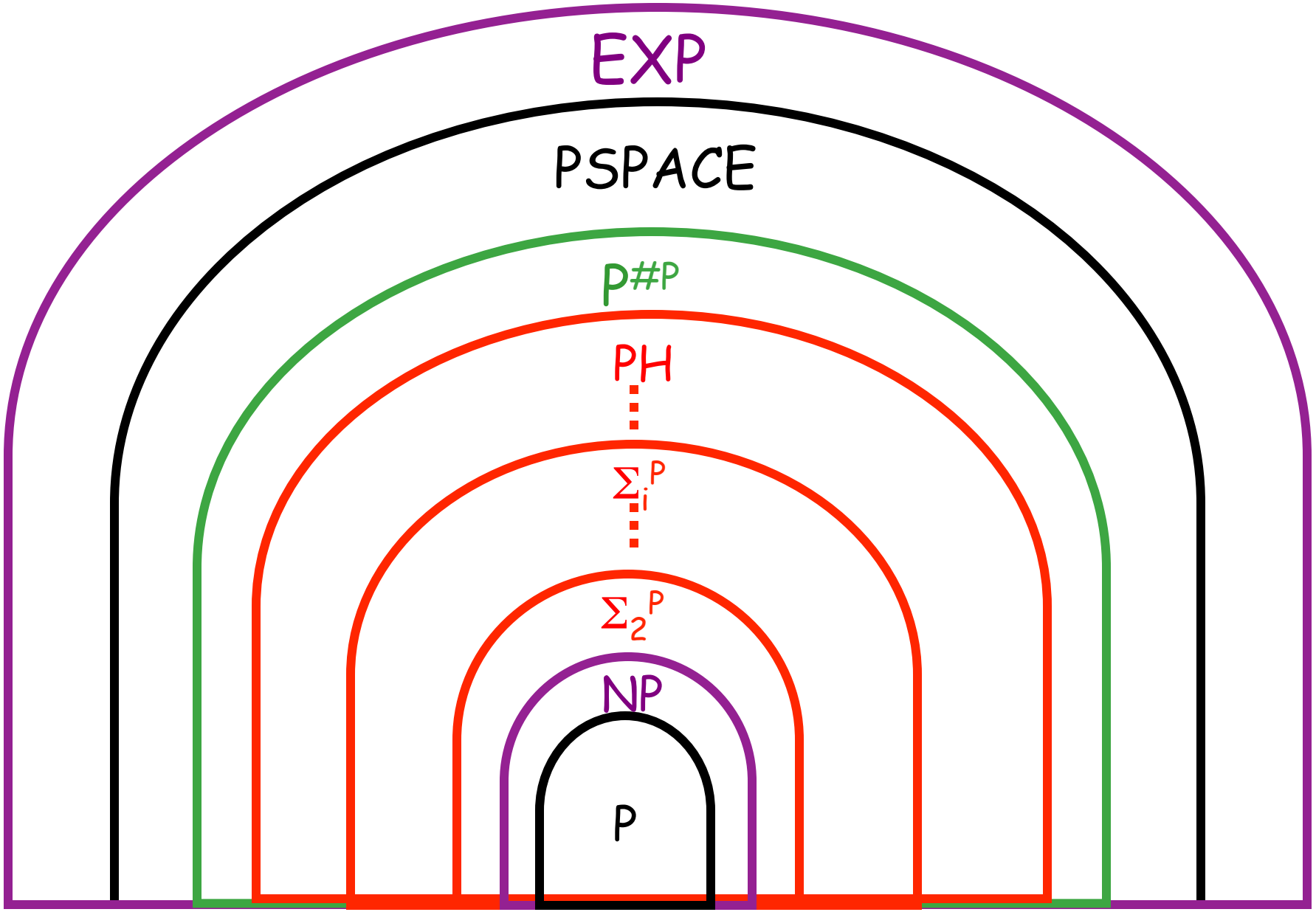
- $x \in L \rightarrow \exists P$: "yes" w.p. $\frac{3}{4}$
- $x \notin L \rightarrow \forall P^*$: "no" w.p. $\frac{3}{4}$

Nontrivial Result

Interactively Provable

Poly-Space Solvable


PSPACE
NP
coNP
RP
coRP
ZPP
P


EXP
PSPACE
P#P
PH
Σ_i^P
Σ_2^P
NP
P