Computer Science 468/568
Homework #7, due in class Thursday, April 5, 2018.

1. (10 points) Please list any persons (including course staff) you talked with about this assignment and cite any resources (other than the textbook) you consulted in connection with this assignment (with enough information that another person could find the materials you cite).

2. (30 points) A probabilistic Turing machine can easily select one out of 2^k alternatives using its random choices, but 1 out of 3 requires a little more thought. Prove that there is a polynomial time probabilistic Turing machine M that takes as input a number n in unary and outputs one of the four strings 00, 01, 10, or 11 and halts. The probability of output 11 should be at most 2^{-n}, and the probabilities of the other three outputs should be exactly equal.

3. (30 points) Suppose that $L \subseteq \{0,1\}^*$ and M is a probabilistic polynomial time Turing machine such that for every $x \in \{0,1\}^*$, if $x \in L$ then $\Pr[M(x) = 1] \geq 1/(|x| + 1)$ and if $x \notin L$ then $\Pr[M(x) = 1] = 0$. Prove that $L \in \mathbf{RP}$ by showing that there is another probabilistic polynomial time Turing machine M' such that for every $x \in \{0,1\}^*$, if $x \in L$ then $\Pr[M'(x) = 1] \geq 2/3$ and if $x \notin L$ then $\Pr[M'(x) = 1] = 0$.

4. (30 points) Prove that $\mathbf{ZPP} = \mathbf{RP} \cap \mathbf{coRP}$. (This is problem 7.6 in the text.)

5. (10 points) This problem is required of students enrolled in CPSC 568, but not of students enrolled in CPSC 468.

   Write a brief (at most one paragraph) progress report for your final paper. Recall that a complete preliminary draft is due April 19 in class.