Heuristic Search

CPSC 470/570 – Artificial Intelligence

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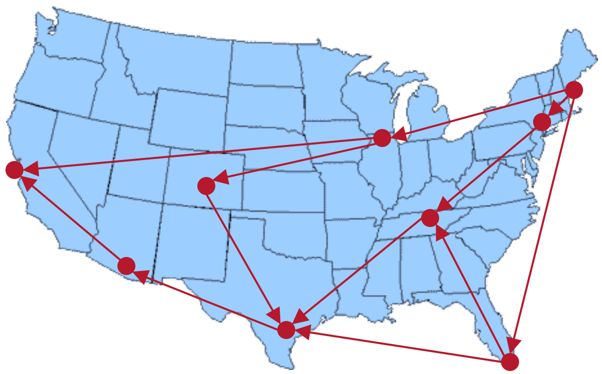
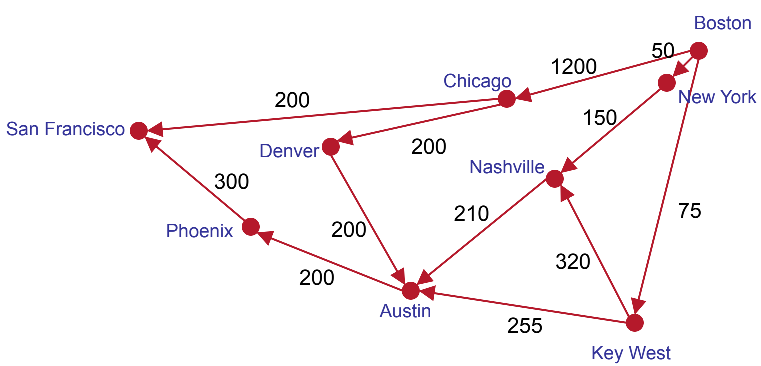
Goal Formulation



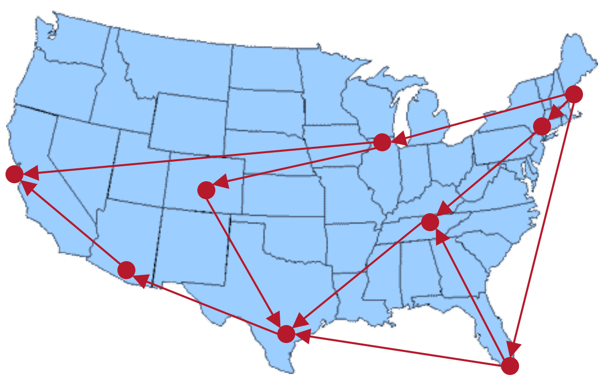


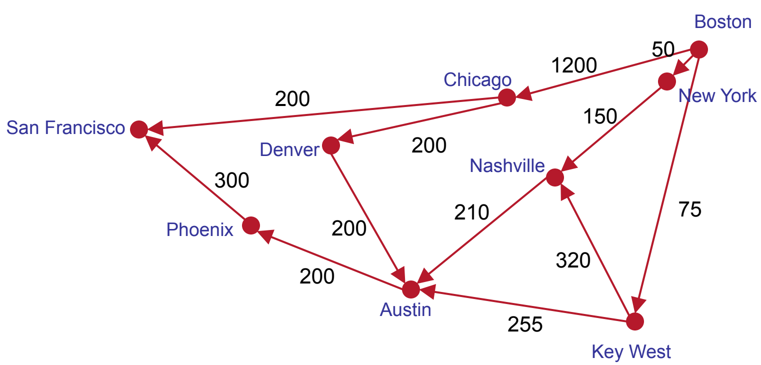
- Well-defined function that identifies both the goal states and the conditions under which to achieve the goal
  - Fly from Boston to San Francisco
  - Quality might depend on
    - Least amount of money
    - Fewest number of transfers
    - Shortest amount of time in the air
    - Shortest amount of time in airports

Problem Formulation

- Well-defined problems
  - Fully observable
  - Deterministic
  - Discrete set of possible actions (operations)
- State space: the set of all states that are reachable from an initial state by any sequence of actions
- Path: sequence of actions leading from one state to another

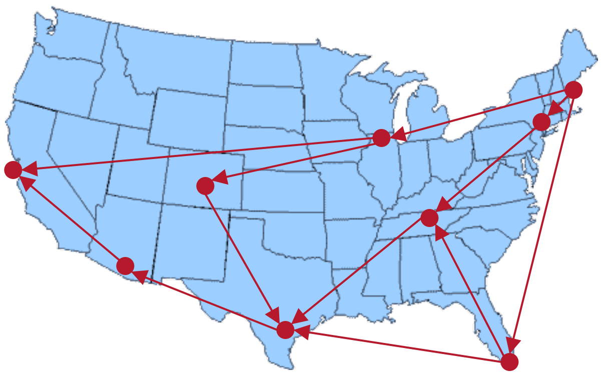
Problem Formulation

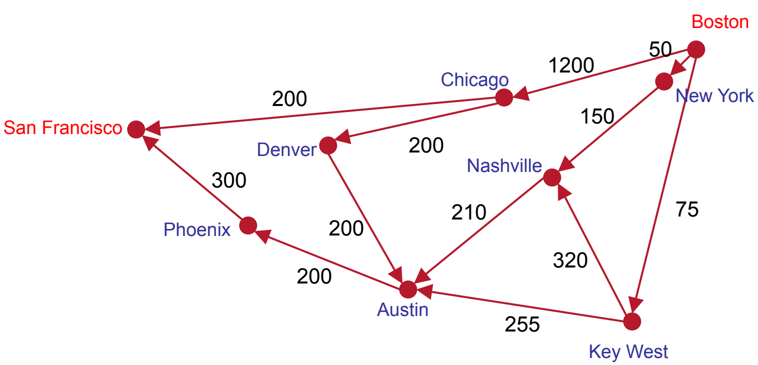


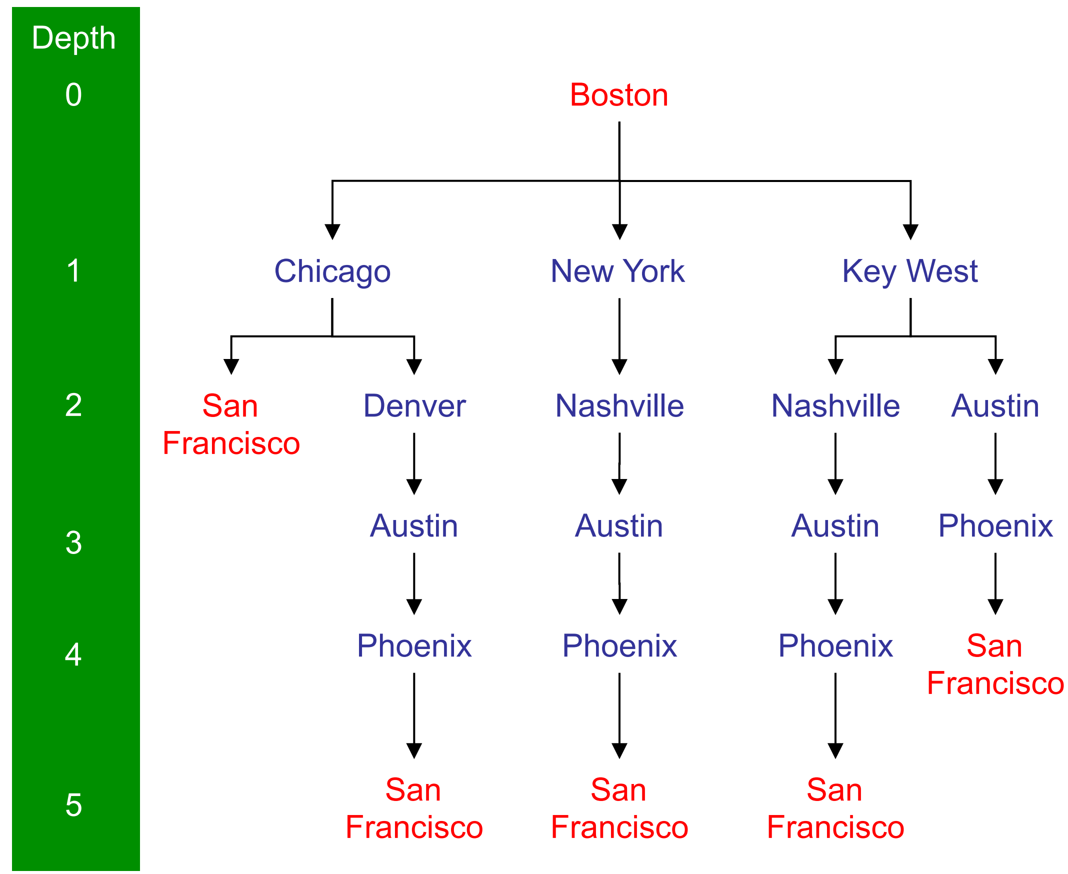


- Goal: spend less $
- State space: flights and their costs
- Path: sequence of flights
- Picking the right level of abstraction
  - Fly from Boston to Chicago
  - Directions to the airport
  - Move left leg 18 inches forward

How to Search:
Generating Sequences and Data Structures







Branching Factor *b*=3

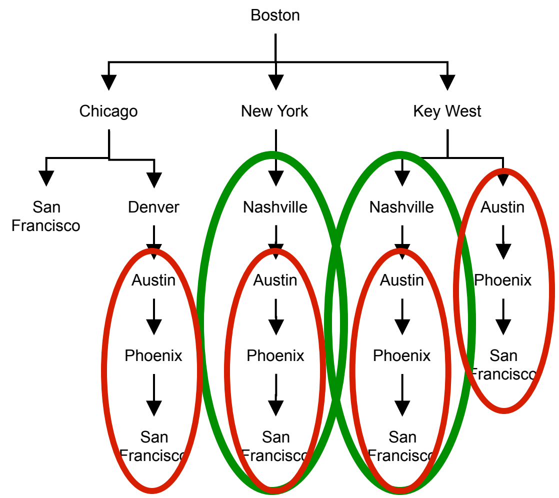
Measuring Performance

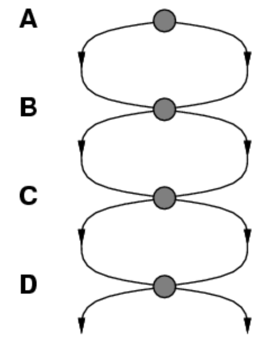
- Completeness: is the strategy guaranteed to find a solution when one exists?
- Time Complexity: how long does it take to find a solution?
- Space Complexity: how much memory does it require to perform the search?
- Optimality: Does the strategy find the best-quality solution when more than one solution exists?

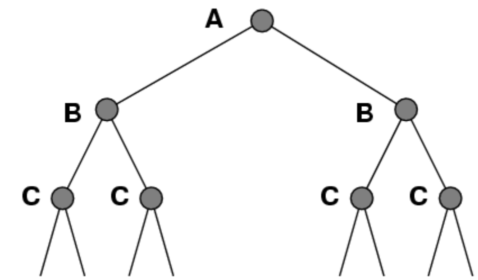
Types of Blind Search

- Breadth-First Search
- Depth-First Search
- Depth Limited Search
- Iterative Deepening Search
- Bi-directional Search

Improving Blind Search: Avoiding Repeated States



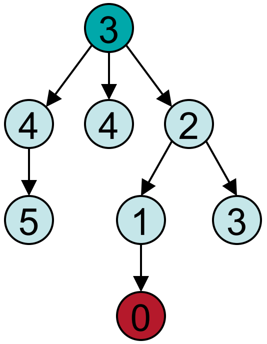


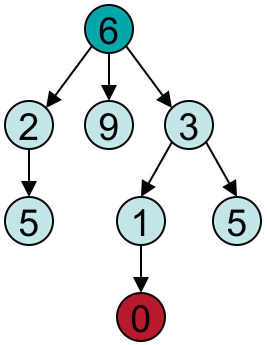


- Simple caching could be used to store the expected values of sub-trees.
  - Must maintain a table of all visited states and the result
- Change the rules for generating the tree
  - Do not generate repeated states
  - Do not generate paths with cycles

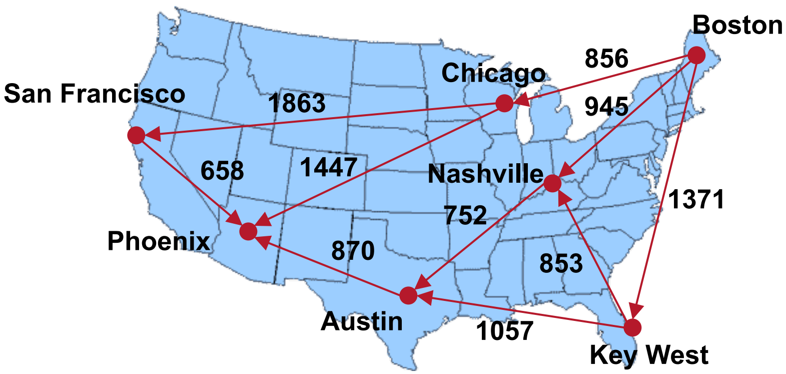
Heuristic Functions

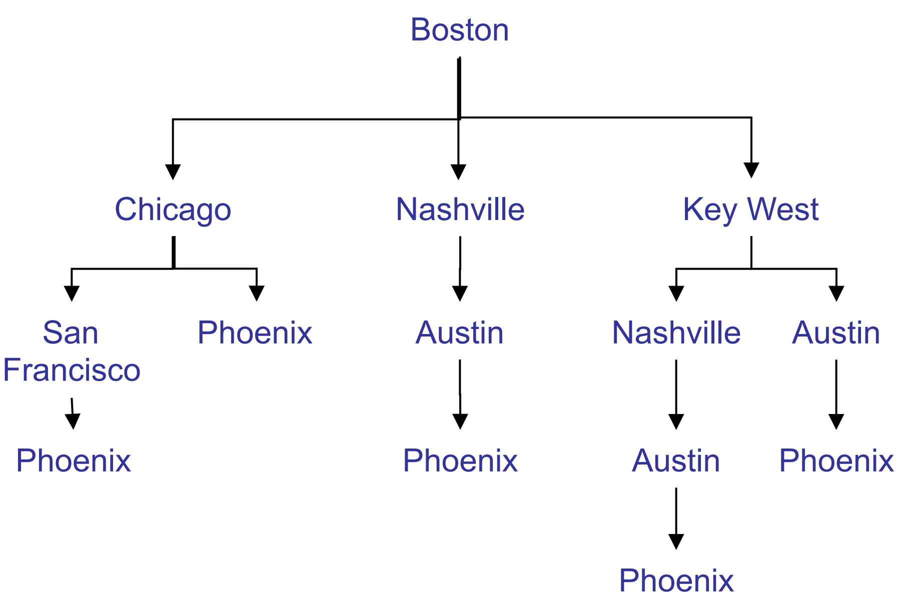
- These techniques are all still brute-force
- Can we do anything more intelligent?
- If we could identify an *evaluation function*, which described how valuable each state was in obtaining the goal, then we could simply always choose to expand the leaf node with the best value.
- A *heuristic function* is an inexact estimate of the evaluation function.





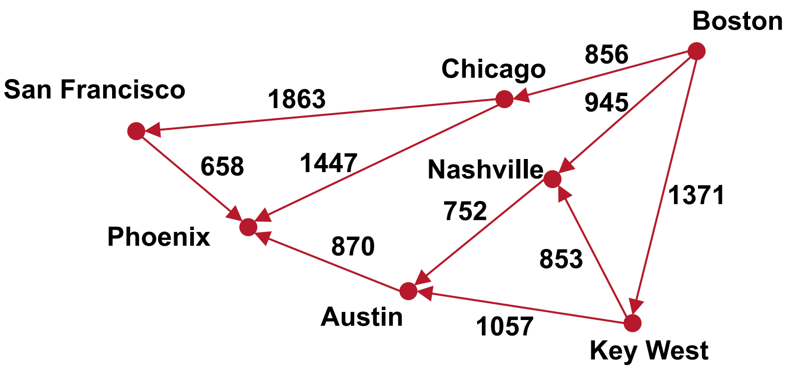
Greedy Best-First Search





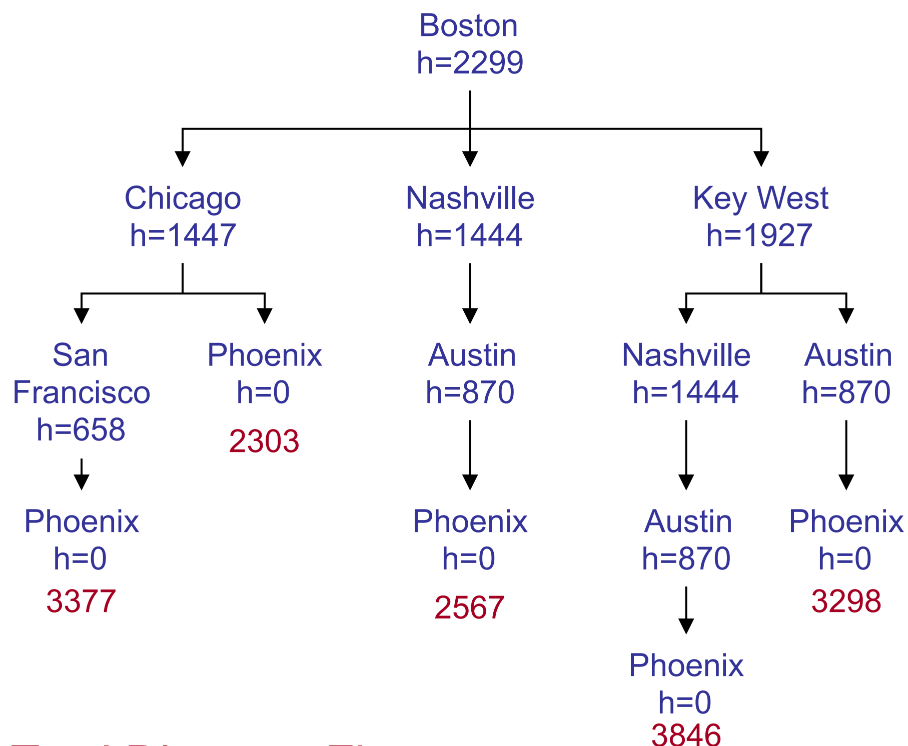
- Rely on a heuristic function to determine which node to expand
- Better name is "best-guess-first" search
- Airline example
  - Find the shortest path from Boston to Phoenix

Greedy Best-First-Search



	Straight Line Distance to Phoenix
Boston	2299
Chicago	1447
Nashville	1444
Key West	1927
Austin	870
San Francisco	658

- Minimize estimated cost to reach a goal (in this case, the distance to Phoenix)

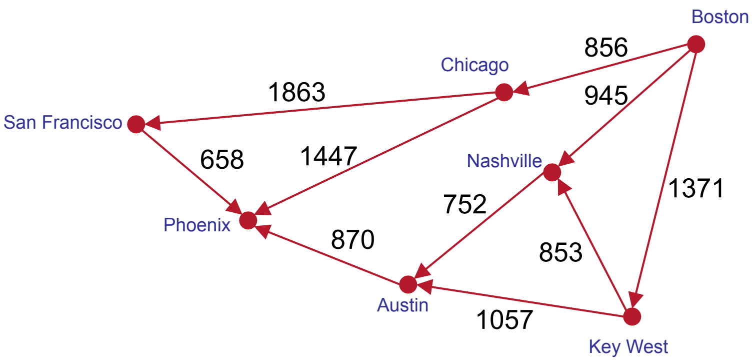


Total Distance Flown

Greedy Best-First-Search

- Optimal?
  - No, as the previous example demonstrated
- Complete?
  - No, just as depth first search
- Worst-case time complexity?
  - $O(b^m)$ where b=branch factor, m=max. depth
- Worst-case space complexity?
  - Same as time complexity… entire tree kept in memory
- Actual time/space complexity
  - Depends on the quality of the heuristic function

A* Search



	Distance to Phoenix
Boston	2299
Chicago	1447
Nashville	1444
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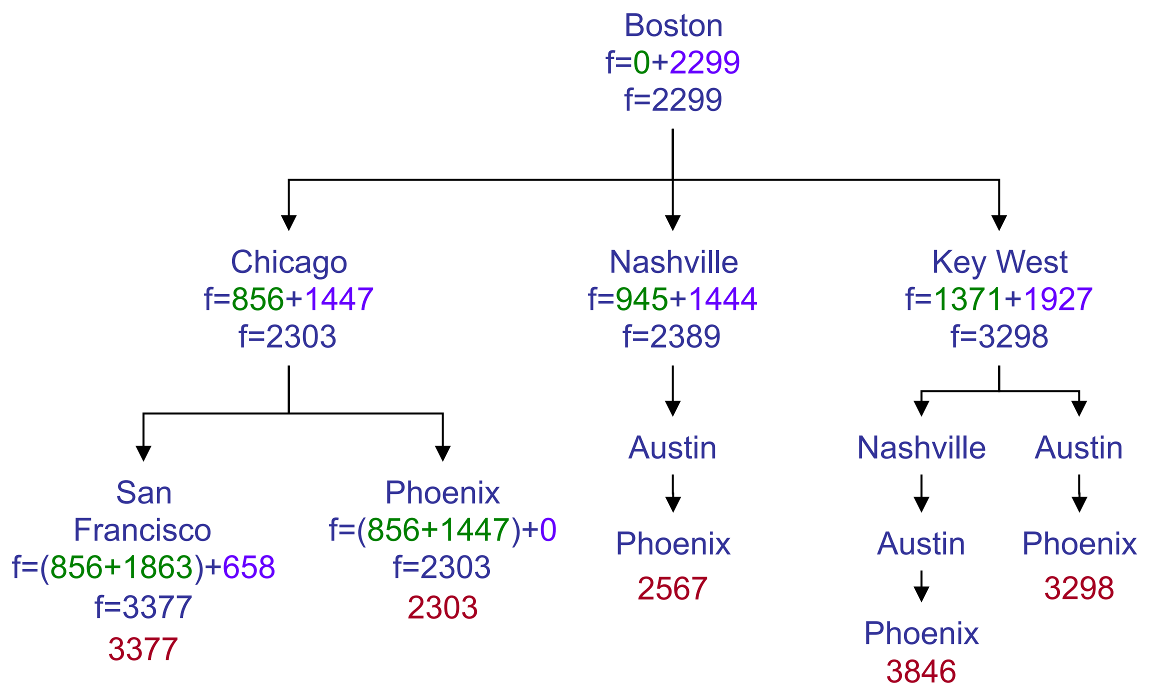
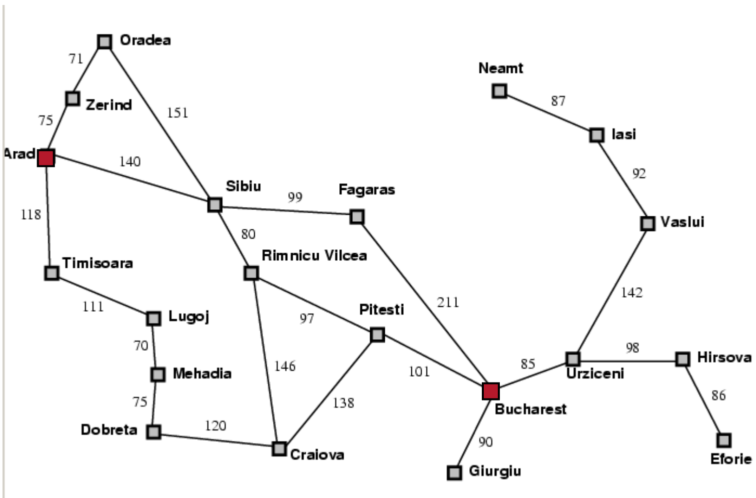
- Combine Greedy search with Uniform Cost Search
- Minimize the total path cost (f) =
  actual path so far (g) +
  estimate of future path to goal (h)



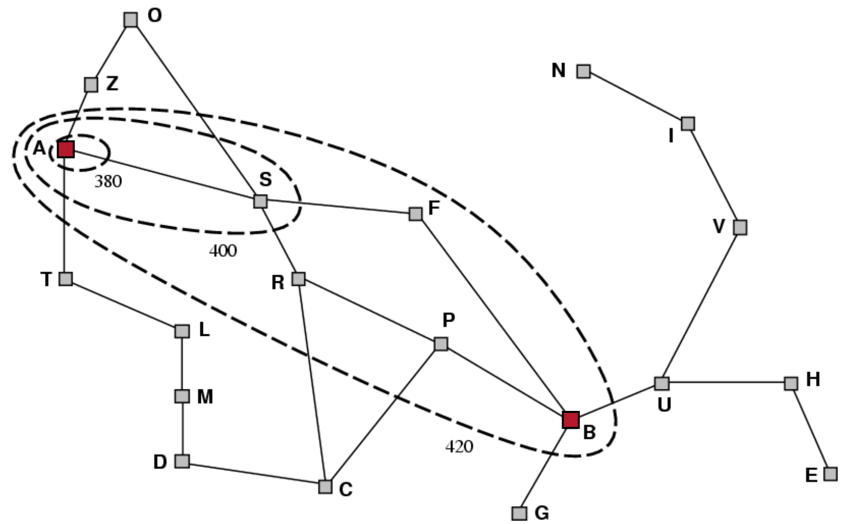
Total Distance Flown

How does A* Search Work?



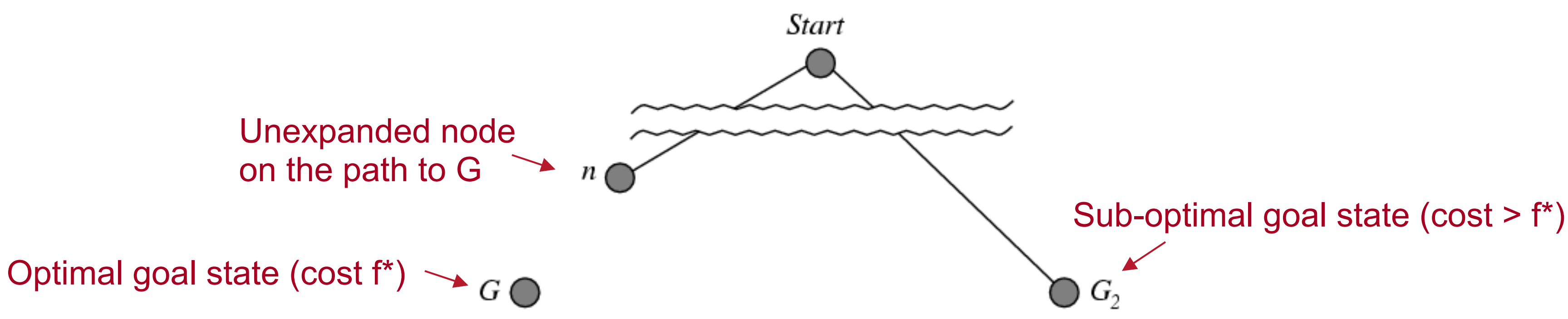
Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374



- The heuristic function *h* must be *admissible*
  - It must never over-estimate the cost to reach the goal
- Most obvious heuristics are *monotonic*
  - If the total path cost is non-monotonic as you move down the tree, you can substitute a monotonic function based on the parent
- Allows the above contour interpretation

Proving the Optimality of A*



- Assume that G_2 has been chosen for expansion over *n*
- Because *h* is admissible

  $f^* \geq f(n)$
- If *n* is not chosen for expansion over G_2, we must have

  $f(n) \geq f(G_2)$
- Combining these, we get

  $f^* \geq f(G_2)$
- However, this violates our assertion that G_2 is sub-optimal
- Therefore, A* never selects a sub-optimal goal for expansion

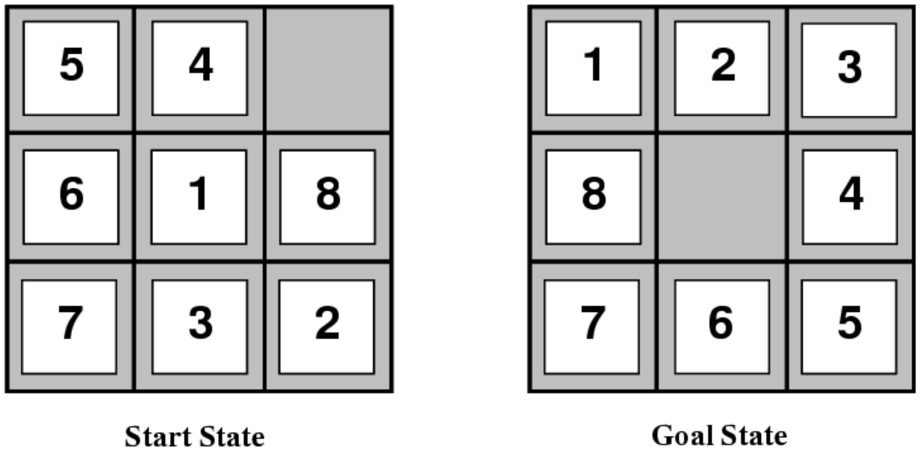
Completeness of A*

- A* expands nodes in order of increasing *f*
- When would a solution not be found?
  - Node with an infinite branching factor
  - A path with a finite path cost but an infinite number of nodes
- A* is complete when
  - There is a finite branching factor
  - Every operator costs at least some positive ε

Complexity of A*

- Computation time is limited by the quality of the heuristic function (but is still exponential)
  - Issue #1 : Choosing the right heuristic function can have a large impact
- More serious problem is that all generated nodes need to be kept in memory
  - Issue #2 : Can we limit the memory requirements?

Issue #1: Choosing a Heuristic Functions



- Must be admissible (never over-estimate)
- Heuristics for the 8-Puzzle
  - *h1* = number of tiles in the wrong position
  - *h2* = sum of the distances of the tiles from their goal positions (city block distance)

Effect of Heuristic Accuracy on Performance in the 8-puzzle

	Search Cost			Effective Branching Factor		
d	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

- Compare iterative-deepening with A* using
  h1 (# misplaced tiles) and *h2* (city block distance)
- Effective branching factor b*
  - Number of expanded nodes = $1 + b^* + (b^*)^2 + \ldots + (b^*)^{depth}$
  - b* remains relatively constant across many measurements
- Always better to use a heuristic with higher values, so long as it does not over-estimate

Issue #2
Limiting Memory Utilization

- If we can maintain a bound on the memory, we might be willing to wait for a solution
- Two techniques for Memory Bounded Search:
  - Iterative deepening A* (IDA*)
  - Recursive Best-First-Search (RBFS)

Iterative Deepening A* Search (IDA*)

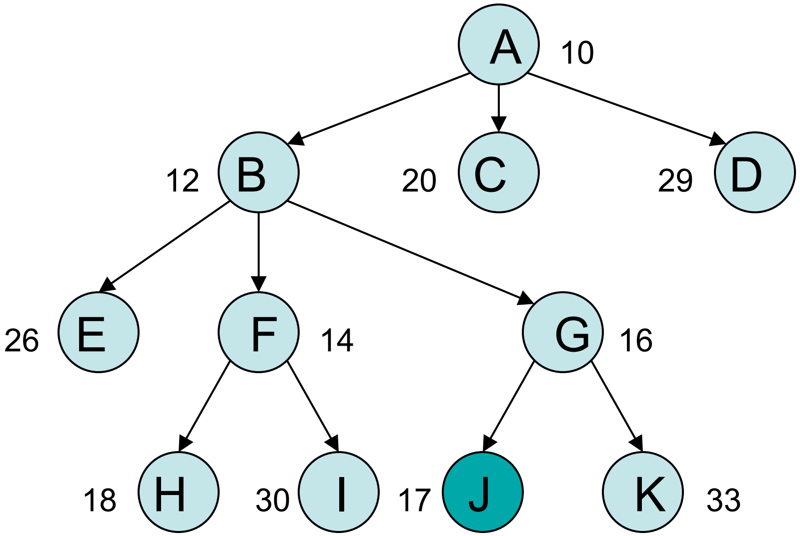
- Each iteration is a depth-first search with a limit based on *f* rather than on depth
- Complete and optimal (with same caveats as A*)
- Requires space proportional to the longest path that it explores
- Can have competitive time complexity, since the overhead of maintaining the nodes in memory is greatly reduced

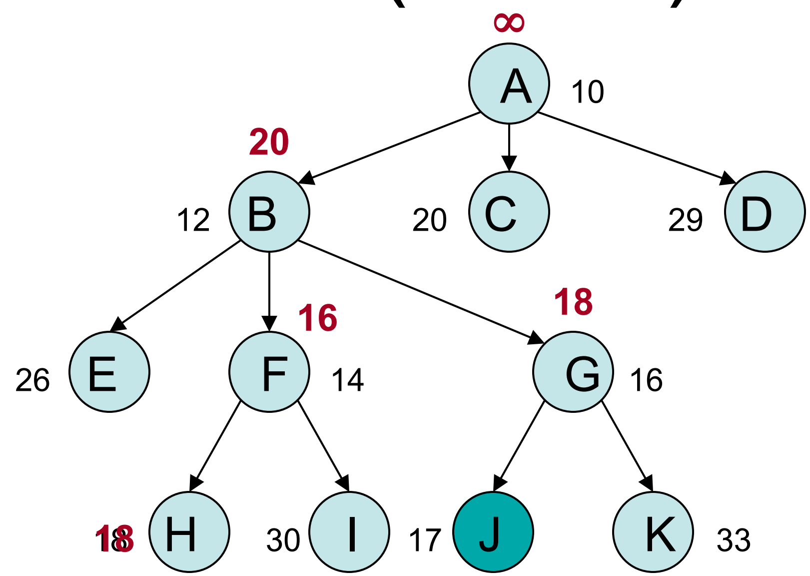
Problems with IDA*

- In the TSP, different heuristic function value for each state
- Each contour contains only one additional node
- If A* expands N nodes, the IDA* will expand

  $1+2+3+4+\ldots+N = O(N^2)$ nodes
- If N is too large for memory, N^2 is too long to wait
- Runs into problems because it recalculates every node

Recursive Best-First Search (RBFS)





- total path cost (f) = actual path so far (g) + heuristic estimate of future path to goal (h)
- Red values best f-value in an alternate branch

Recursive Best-First Search (RBFS)

- RBFS will
  - be complete given sufficient memory to store the shallowest solution path
  - be optimal if the heuristic function is admissible (and you have enough memory to store the solution)
- Both RBFS and IDA* use not enough memory.
  - Require at most linear space with the depth of the tree

tinyurl.com/yale-robot-study

Play video games with a robot!