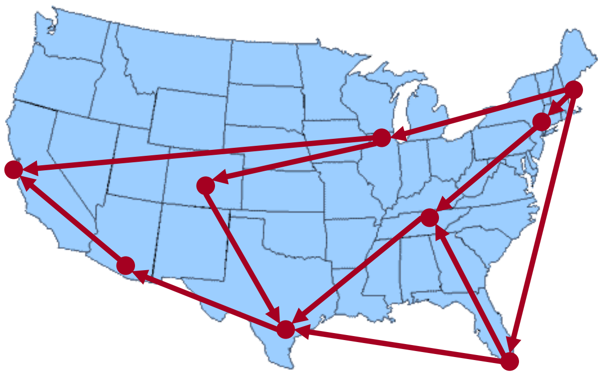
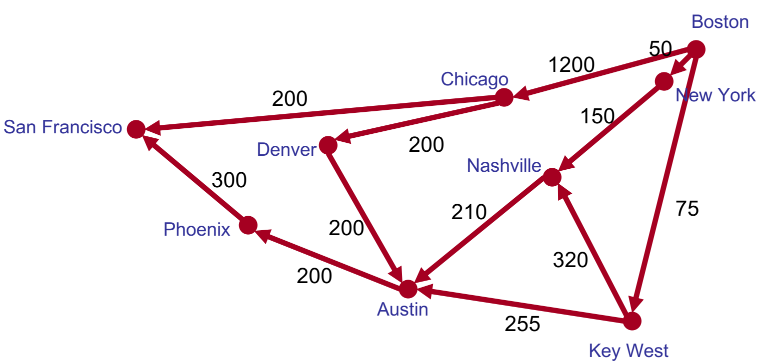
Informed Search

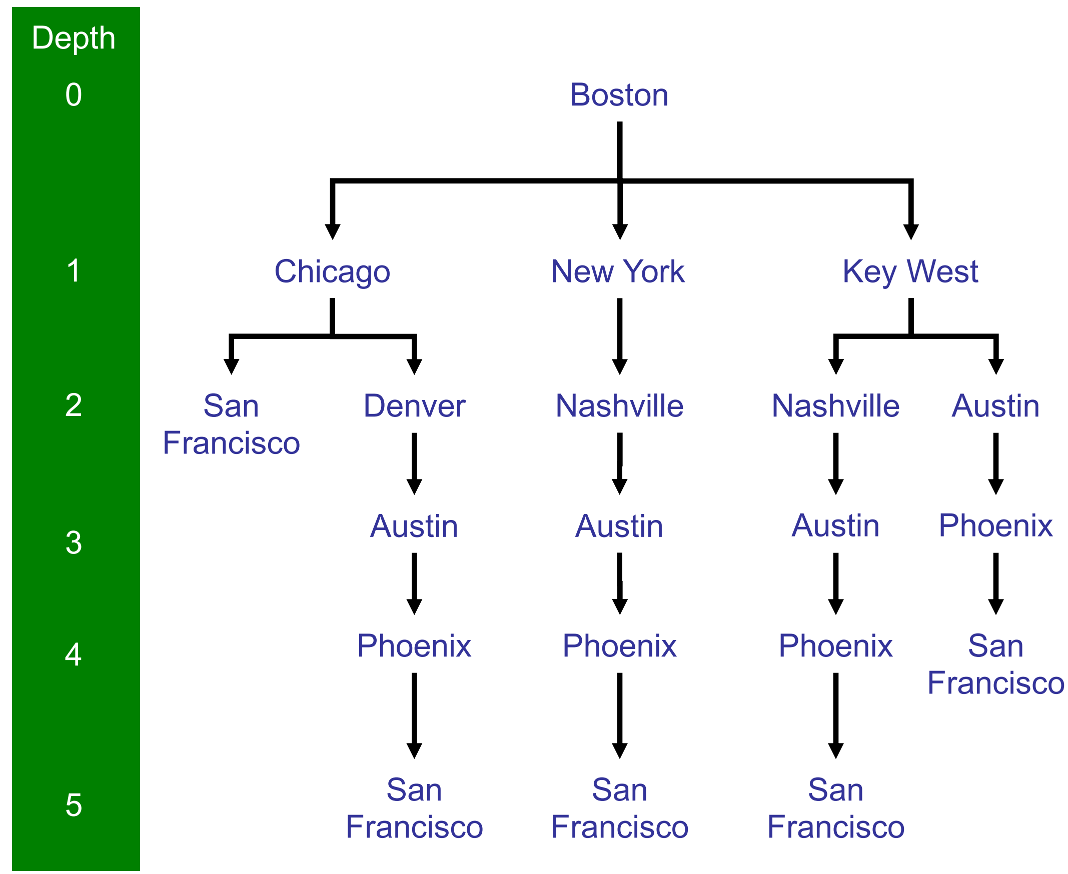
CPSC 470 – Artificial Intelligence

Brian Scassellati

Search as a Problem-Solving Technique





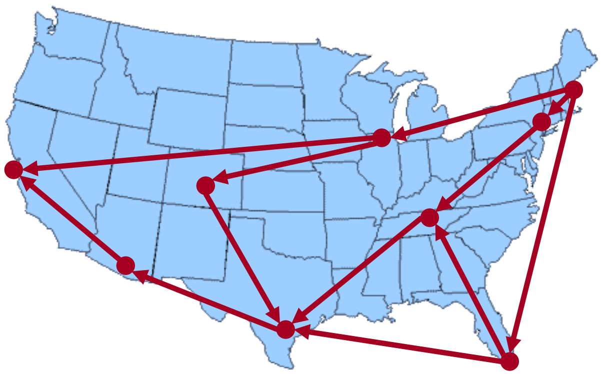


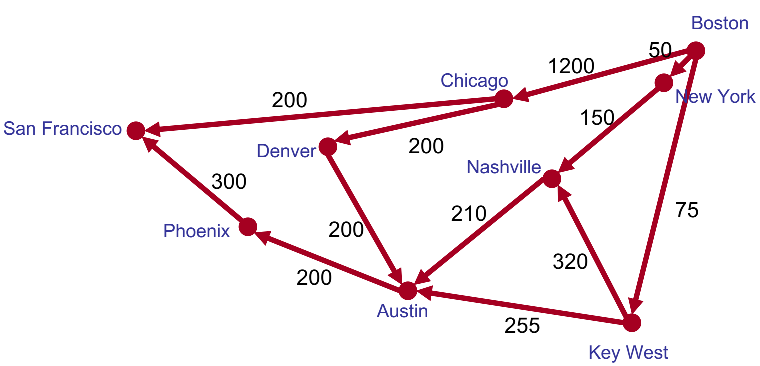
Branching Factor *b*=3

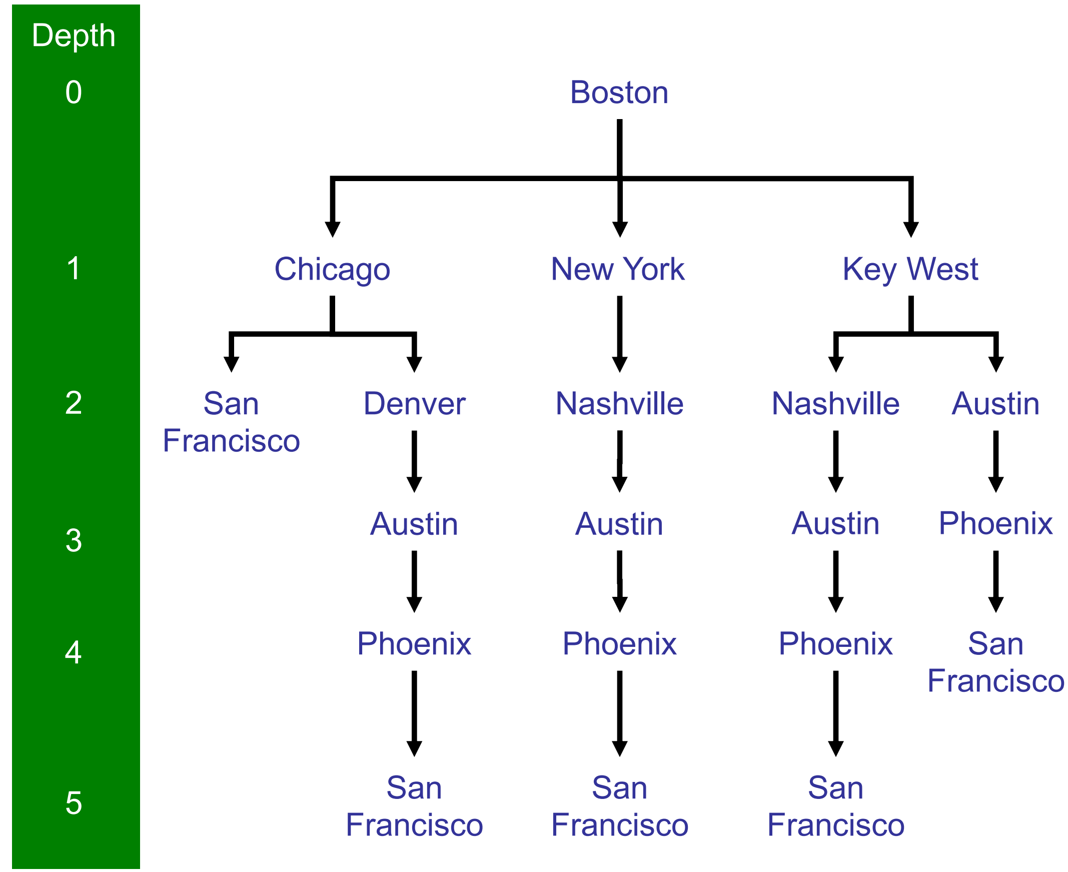
Types of Blind Search

- Breadth-First Search
- Depth-First Search
- Depth Limited Search
- Iterative Deepening Search
- Bi-directional Search

Search as a Problem-Solving Technique

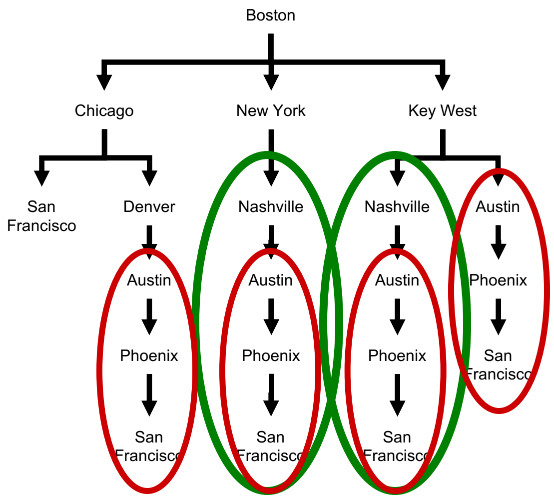


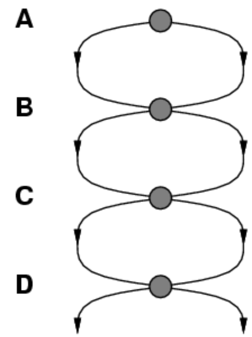


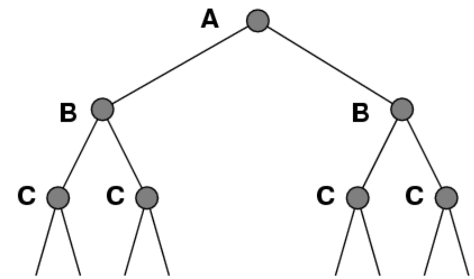


Branching Factor *b*=3

Improving Blind Search: Avoiding Repeated States



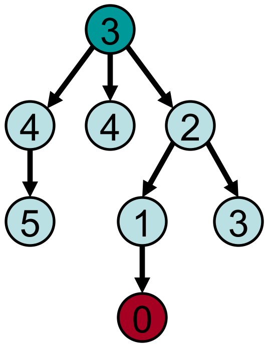




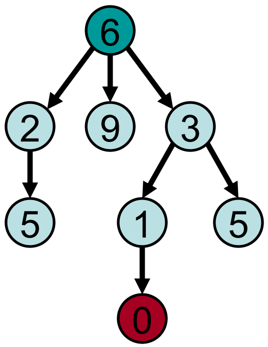
- Simple caching could be used to store the expected values of sub-trees.
  - Must maintain a table of all visited states and the result
- Change the rules for generating the tree
  - Do not generate repeated states
  - Do not generate paths with cycles

Heuristic Functions

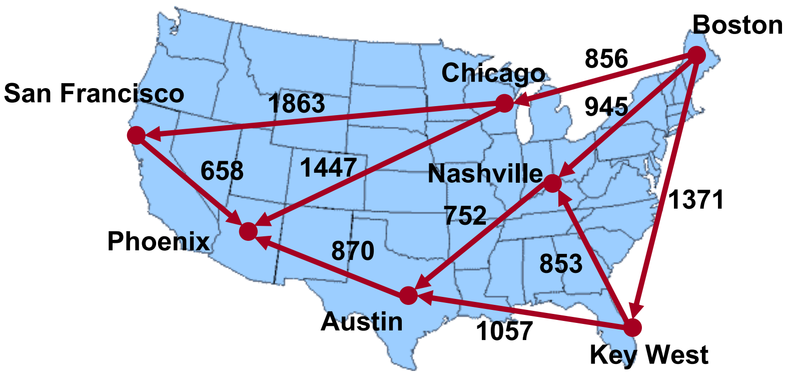
- These techniques are all still brute-force
- Can we do anything more intelligent?
- If we could identify an *evaluation function*, which described how valuable each state was in obtaining the goal, then we could simply always choose to expand the leaf node with the best value.

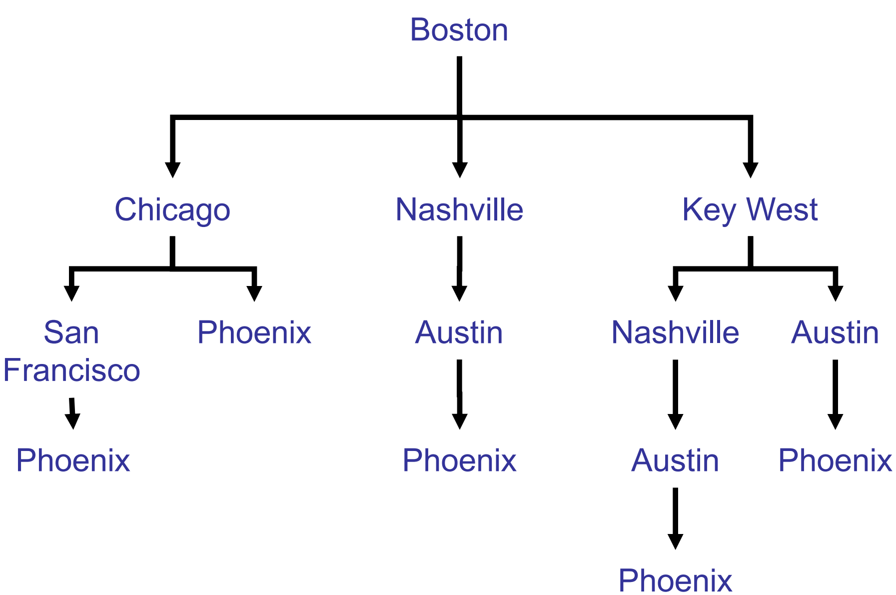


- A *heuristic function* is an inexact estimate of the evaluation function.



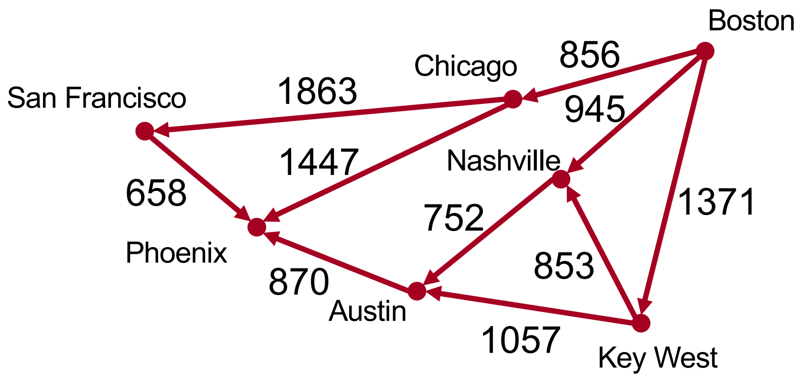
Greedy Best-First Search





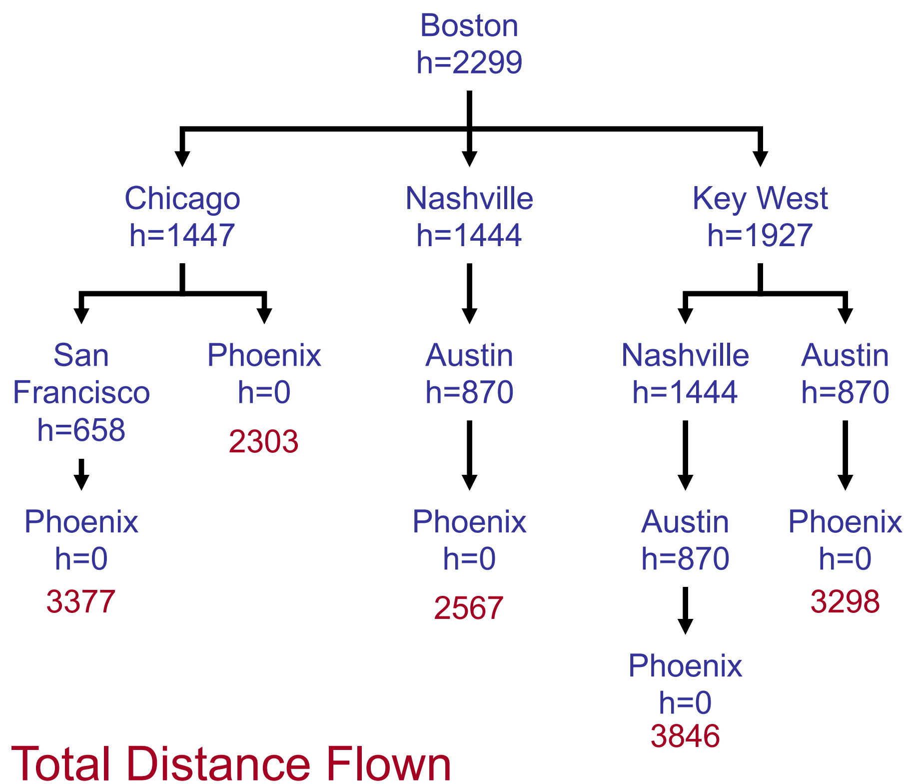
- Rely on a heuristic function to determine which node to expand
- Better name is "best-guess-first" search
- Airline example
  - Find the shortest path from Boston to Phoenix

Greedy Best-First-Search



- Minimize estimated cost to reach a goal (in this case, the distance to Phoenix)

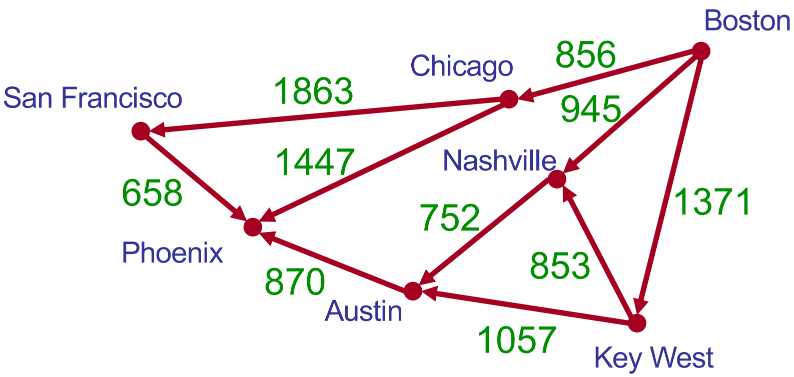
	Distance to Phoenix
Boston	2299
Chicago	1447
Nashville	1444
Key West	1927
Austin	870
San Francisco	658



Greedy Best-First-Search

- Optimal?
  - No, as the previous example demonstrated
- Complete?
  - No, just as depth first search
- Worst-case time complexity?
  - $O(b^m)$ where b=branch factor, m=max. depth
- Worst-case space complexity?
  - Same as time complexity… entire tree kept in memory
- Actual time/space complexity
  - Depends on the quality of the heuristic function

A* Search



- Combine Greedy search with Uniform Cost Search
- Minimize the total path cost (f) =
  actual path so far (g) +
  estimate of future path to goal (h)

	Distance to Phoenix
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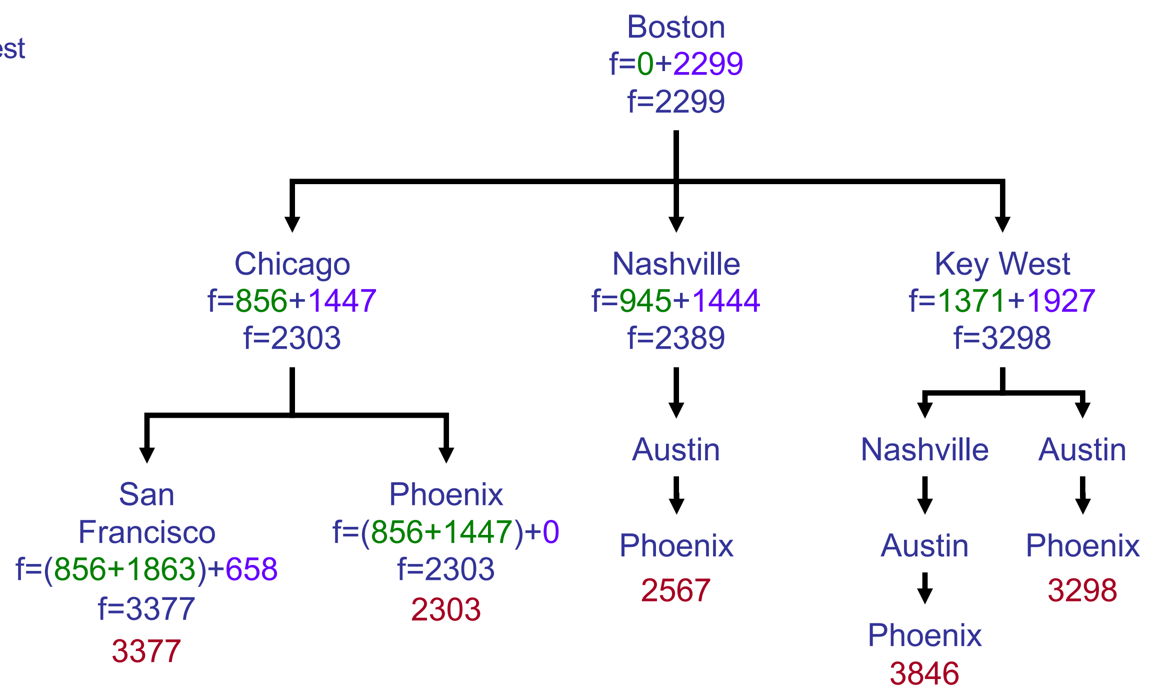
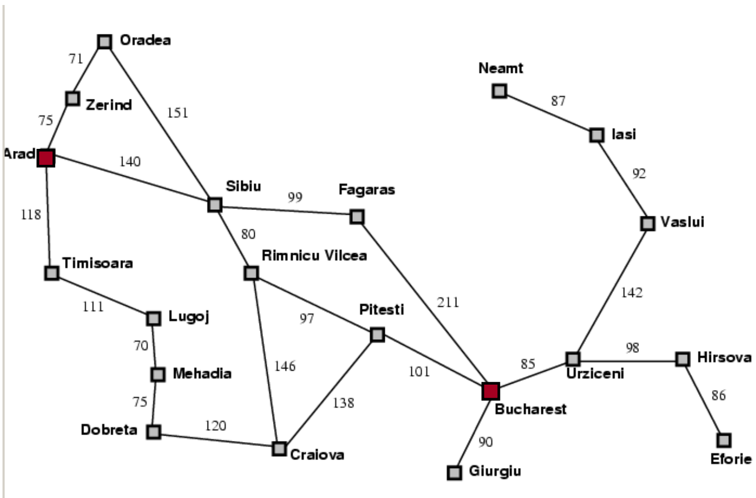
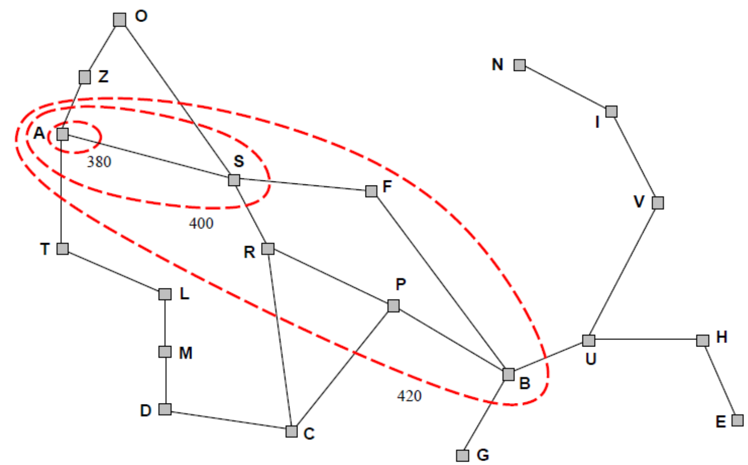


Total Distance Flown

How does A* Search Work?

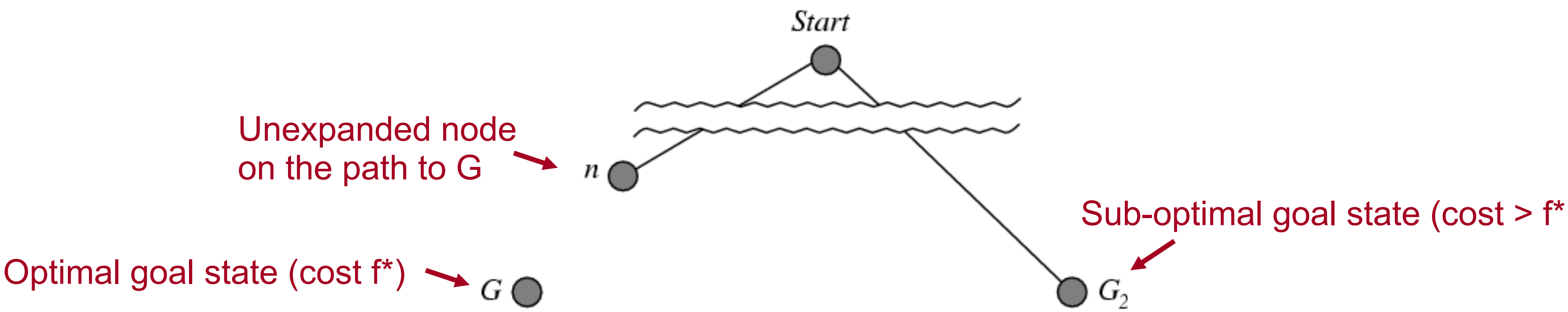


Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374



- A* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Requires that the heuristic function *h* must be *admissible*
  - It must never over-estimate the cost to reach the goal

Proving the Optimality of A*



- Assume that G_2 has been chosen for expansion over *n*
- Because *h* is admissible

  $f^* \geq f(n)$
- If *n* is not chosen for expansion over G_2, we must have

  $f(n) \geq f(G_2)$
- Combining these, we get

  $f^* \geq f(G_2)$
- However, this violates our assertion that G_2 is sub-optimal
- Therefore, A* never selects a sub-optimal goal for expansion

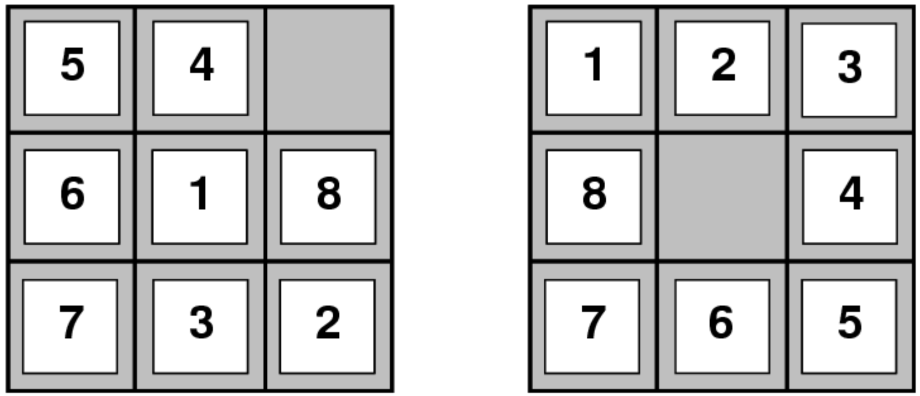
Completeness of A*

- A* expands nodes in order of increasing *f*
- When would a solution not be found?
  - Node with an infinite branching factor
  - A path with a finite path cost but an infinite number of nodes
- A* is complete when
  - There is a finite branching factor
  - Every operator costs at least some positive ε

Complexity of A*

- Computation time is limited by the quality of the heuristic function (but is still exponential)
  - Issue #1 : Choosing the right heuristic function can have a large impact
- More serious problem is that all generated nodes need to be kept in memory
  - Issue #2 : Can we limit the memory requirements?

Issue #1:
Choosing a Heuristic Functions



Start State Goal State

- Must be admissible (never over-estimate)
- Heuristics for the 8-Puzzle
  - *h1* = number of tiles in the wrong position (h1=7)
  - *h2* = sum of the distances of the tiles from their goal positions (*city block / Manhattan distance*)

h2 = 2+3+3+2+4+2+0+2 = 18

Effect of Heuristic Accuracy on Performance in the 8-puzzle

	Search Cost		
d	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6
4	112	13	12
6	680	20	18
8	6384	39	25
10	47127	93	39
12	364404	227	73
14	3473941	539	113
16	–	1301	211
18	–	3056	363
20	–	7276	676
22	–	18094	1219
24	–	39135	1641

- Compare iterative-deepening search (IDS) with A* using
  - *h1* (# misplaced tiles)
  - *h2* (city block distance)
- Always better to use a heuristic with higher values, so long as it does not over-estimate

Issue #2
Limiting Memory Utilization

- If we can maintain a bound on the memory, we might be willing to wait for a solution
- Two techniques for Memory Bounded Search:
  - Iterative deepening A* (IDA*)
  - Recursive Best-First-Search (RBFS)

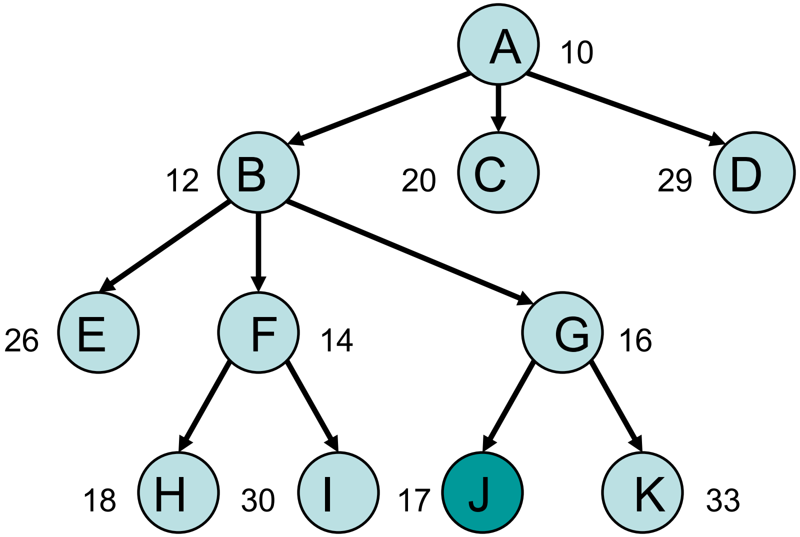
Iterative Deepening A* Search (IDA*)

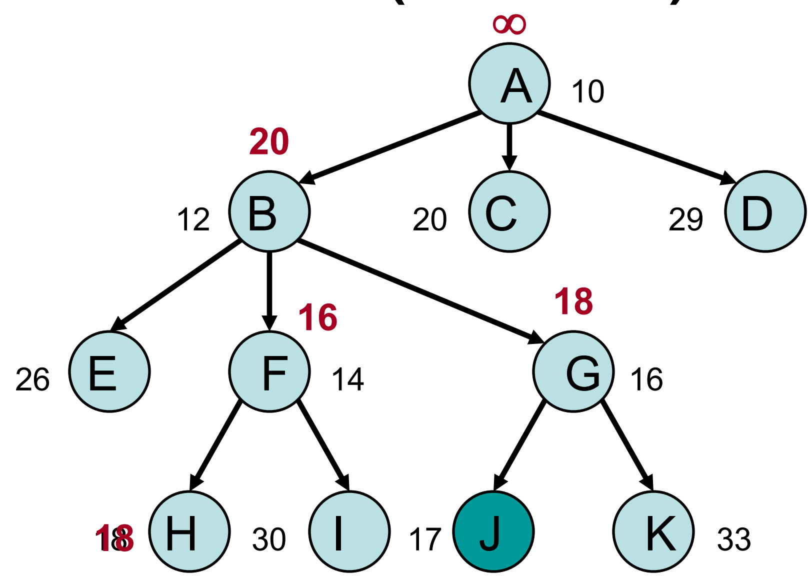
- Each iteration is a depth-first search with a limit based on *f* rather than on depth
- Complete and optimal (with same caveats as A*)
- Requires space proportional to the longest path that it explores
- Can have competitive time complexity, since the overhead of maintaining the nodes in memory is greatly reduced

Problems with IDA*

- In the TSP, different heuristic function value for each state
- Each contour contains only one additional node
- If A* expands N nodes, the IDA* will expand
  1+2+3+4+…+N = $O(N^2)$ nodes
- If N is too large for memory, N^2 is too long to wait
- Runs into problems because it recalculates every node

Recursive Best-First Search (RBFS)





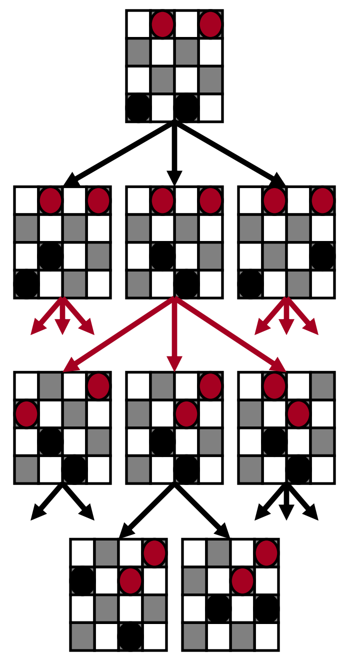
- total path cost (f) = actual path so far (g) + heuristic estimate of future path to goal (h)
- Red values best f-value in an alternate branch

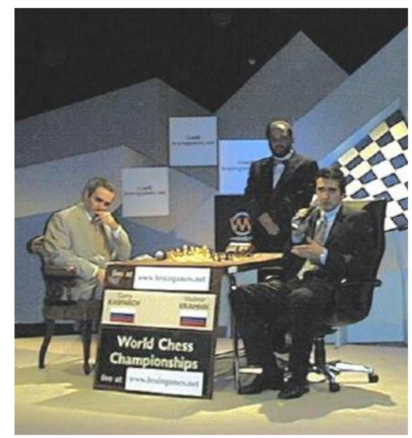
Recursive Best-First Search (RBFS)

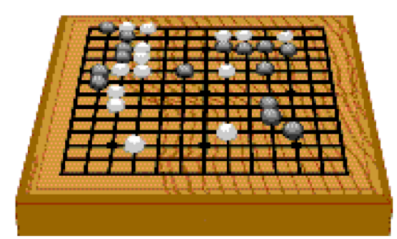
- RBFS will
  - be complete given sufficient memory to store the shallowest solution path
  - be optimal if the heuristic function is admissible (and you have enough memory to store the solution)
- Both RBFS and IDA* use not enough memory.
  - Require at most linear space with the depth of the tree

Coming Up Next…

- Can you search without building a tree?
- What happens when you don’t get to make the choice at each level of the search space?
- Game Playing
  - Minimax
  - Alpha-beta pruning







Administrivia

- Problem Set 1 is now out
- Next week:
  - Monday: Game Playing
  - Wednesday: No class
  - Friday: Guest lecture – Dragomir Radev