Inference

CPSC 470 – Artificial Intelligence

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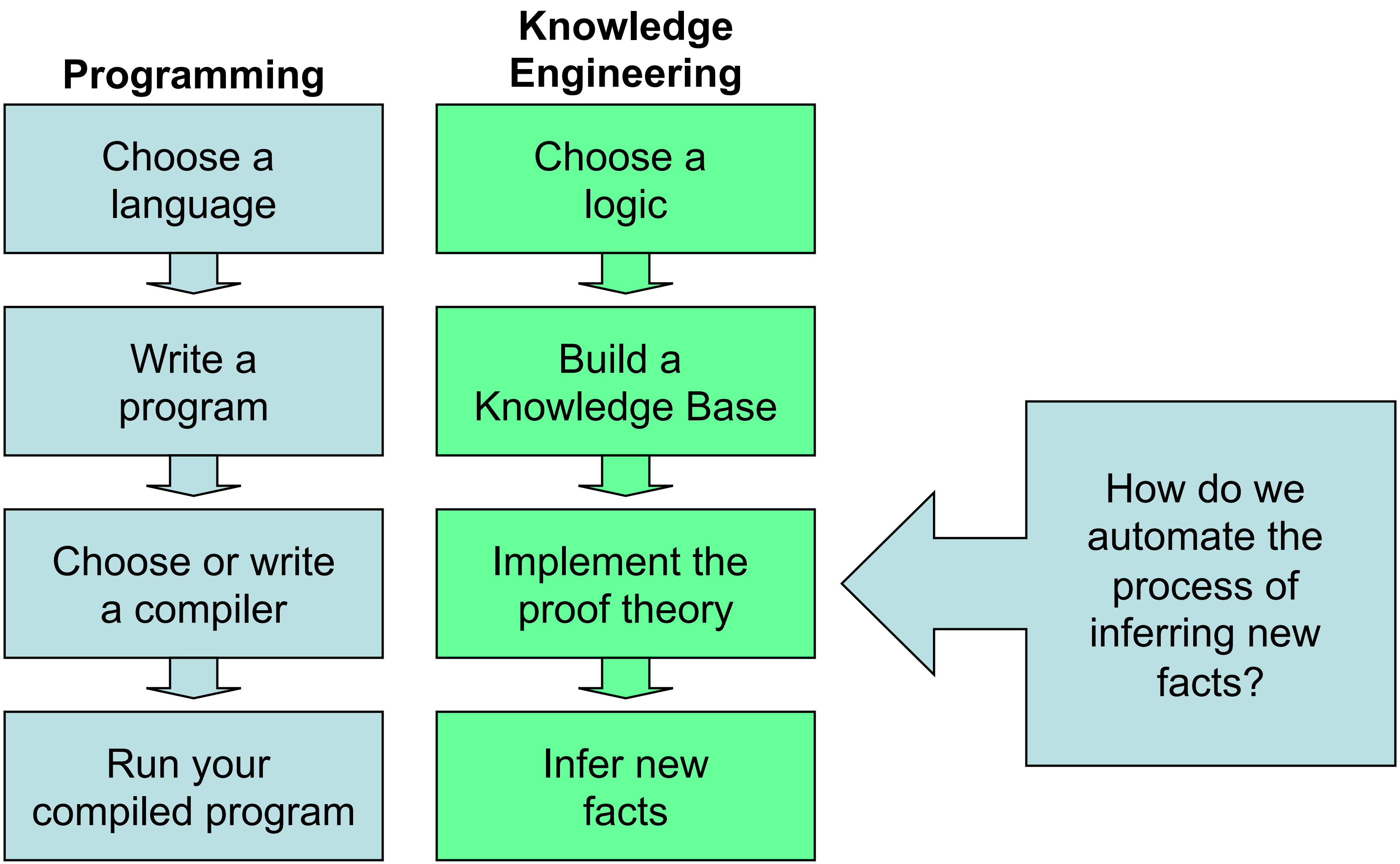
Analogies to Programming



Review of Inference in Propositional Logic

Inference Rules for Propositional Logic

- Modus Ponens (Implication-Elimination)
  - From an implication and its premise, infer conclusion

$$\frac{\alpha \Rightarrow \beta \ , \alpha}{\beta}$$

- And-Elimination
  - From a conjunction, you can infer any conjunct

$$\frac{\alpha_1 \wedge \alpha_2 \wedge \alpha_3 \wedge \ldots \wedge \alpha_n}{\alpha_i}$$

Inference Rules for Propositional Logic

- And-Introduction
  - From a list of sentences, you can infer the conjunct

$$\frac{\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n}{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}$$

- Or-Introduction
  - From a sentence, infer its disjunction with anything

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_n}$$

Inference Rules for Propositional Logic

- Double-Negative Elimination
  - From a double negation, infer the positive sentence

$$\frac{\neg\neg\alpha}{\alpha}$$

- Unit Resolution
  - From a disjunction in which one is false, then you can infer the other is true

$$\frac{\alpha \vee \beta \;,\;\; \neg\beta}{\alpha}$$

Inference Rules for Propositional Logic

- Resolution
  - Since beta cannot be both true and false, one of the disjuncts must be true

$$\frac{\alpha \vee \beta \ , \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

  - Implication is transitive

$$\frac{\alpha \Rightarrow \beta \quad , \quad \beta \Rightarrow \gamma}{\alpha \Rightarrow \gamma}$$

First-Order Logic Requires Additional Rules of Inference

Inference involving Quantifiers

- Remove variables using a substitution function

  Subst(θ,α) : apply the binding list θ to the sentence α

  Subst({x/Tom, y/Jerry}, Chases(x,y))= Chases(Tom,Jerry)

Inference involving Quantifiers

- Universal Elimination
  - For any sentence α, variable *v*, and ground term *g*

$$\frac{\forall v\ \alpha}{Subst(\{v/g\},\alpha)}$$

  - For example, from ∀x Likes(x,FrenchFries) we can substitute {x/Ben} to conclude Likes(Ben,FrenchFries)

Inference involving Quantifiers

- Existential Elimination
  - For any sentence α, variable *v*, and constant symbol *k* that does not appear elsewhere

$$\frac{\exists v \ \alpha}{Subst(\{v/k\},\alpha)}$$

  - For example, ∃x Kill(x,Victim) we can infer Kill(Murderer,Victim)
  - ONLY works when Murderer does not already appear in the KB

Inference involving Quantifiers

- Existential Introduction
  - For any sentence α, variable *v* that does **not** occur in α and ground term *g* that does appear in α

$$\frac{\alpha}{\exists v \; Subst(\{g / v\}, \alpha)}$$

  - For example, from Speaks(Jim,German) we can infer ∃x Speaks(x,German)

An Example Proof

It is a crime for an American to sell alcohol to a minor. Jimmy, a minor, has some beer. All of Jimmy's beer was sold to him by Nathan, an American.

Prove that Nathan is a criminal.

Goal: Criminal(Nathan)

1. $\forall$x,y,z American(x) $\wedge$ Alcohol(y) $\wedge$ Minor(z) $\wedge$ Sells(x,y,z) $\Rightarrow$ Criminal(x)
2. Minor(Jimmy)
3. $\exists$x Owns(Jimmy,x) $\wedge$ Beer(x)
4. $\forall$x Owns(Jimmy,x) $\wedge$ Beer(x) $\Rightarrow$ Sells(Nathan,x,Jimmy)
5. American(Nathan)
6. $\forall$x Beer(x)$\Rightarrow$Alcohol(x)

#	FOPC statement	Reasoning
	Criminal(Nathan)	-- GOAL --
1	$\forall$x,y,z American(x) $\wedge$ Alcohol(y) $\wedge$ Minor(z) $\wedge$ Sells(x,y,z) $\Rightarrow$ Criminal(x)	given
2	Minor(Jimmy)	given
3	$\exists$x Owns(Jimmy,x) $\wedge$ Beer(x)	given
4	$\forall$x Owns(Jimmy,x) $\wedge$ Beer(x) $\Rightarrow$ Sells(Nathan,x,Jimmy)	given
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4	∀x Owns(Jimmy,x) ∧ Beer(x) ⇒ Sells(Nathan,x,Jimmy)	given
5	American(Nathan)	given
6	∀x Beer(x)⇒Alcohol(x)	given
7	Owns(Jimmy, B1) ∧ Beer(B1)	Existential elim on 3

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8a 8b	Owns(Jimmy, B1) Beer(B1)	And-elim on 7

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7	Owns(Jimmy, B1) $\wedge$ Beer(B1)	Existential elim on 3
8a 8b	Owns(Jimmy, B1) Beer(B1)	And-elim on 7
9	Beer(B1) $\Rightarrow$ Alcohol(B1)	Universal elim on 6

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10	Alcohol(B1)	Modus ponens 9, 8b

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7	Owns(Jimmy, B1) ∧ Beer(B1)	Existential elim on 3
8a	Owns(Jimmy, B1)	And-elim on 7
8b	Beer(B1)	
9	Beer(B1) ⇒ Alcohol(B1)	Universal elim on 6
10	Alcohol(B1)	Modus ponens 9, 8b
11	Owns(Jimmy,B1) ∧ Beer(B1) ⇒ Sells(Nathan,B1,Jimmy)	Universal elim on 4

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12	Sells(Nathan,B1,Jimmy)	Modus ponens 11 7

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13	American(Nathan) ∧ Alcohol(B1) ∧ Minor(Jimmy) ∧ Sells(Nathan,B1,Jimmy) ⇒ Criminal(Nathan)	Universal elim on 1 (x3)

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14	American(Nathan) ∧ Alcohol(B1) ∧ Minor(Jimmy) ∧ Sells(Nathan,B1,Jimmy)	And introduction 5, 10, 2, 12

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14	American(Nathan) $\wedge$ Alcohol(B1) $\wedge$ Minor(Jimmy) $\wedge$ Sells(Nathan,B1,Jimmy)	And introduction 5, 10, 2, 12
15	Criminal(Nathan)	Modus ponens 13, 14

Can we perform inference as a search problem?

- Our example proof has 6 initial sentences and 9 additional proof steps
  - Initial state: 6 sentences
  - Operators: ~10 inference rules
    - But can be applied multiple ways
  - Goal: obtain the sentence Criminal(Nathan)
- Branching factor increases as the knowledge base grows
- Universal elimination can have a large branching factor on its own (any ground term can be used)

Unification

- Unification is the process of finding substitutions that match a set of conditions
- The UNIFY algorithm takes two sentences and returns a unifier (a binding list) for them if one exists:

  UNIFY(p,q)=Θ where SUBST(Θ,p)=SUBST(Θ,q)

Unification Examples

- Example using Knows(x,y)

  UNIFY(Knows(John, x), Knows(John, Jane))

  = {x/Jane}

  UNIFY(Knows(John, x), Knows(y, Bill))

  = {x/Bill, y/John}

  UNIFY(Knows(John, x), Knows(y, Mother(y)))

  = {y/John, x/Mother(John)}

  UNIFY(Knows(John, x), Knows(x, Elizabeth))

  = FAILURE

  UNIFY(Knows(John, x_1), Knows(x_2, Elizabeth))

  = {x_1/Elizabeth, x_2/John}

Using Inference Rules

We can use these rules in two ways

- We can generate new inferences from existing sentences to expand the knowledge base
  - Used when a new sentence is added to KB
  - From premises to implications
  - Forward chaining
- We can try to prove a given sentence
  - From implications to premises
  - Backward chaining

Forward Chaining

- Forward-Chaining:
  - Until no rule produces a new assertion
    - For each rule,
      - For each set of possible variable bindings
        - » Instantiate the consequent
        - » If the instantiated consequent is not already asserted, then assert it.
- Data-driven process (not driven toward any particular goal)

Forward Chaining Example

1. $\forall$x,y,z American(x) $\wedge$ Alcohol(y) $\wedge$ Minor(z) $\wedge$ Sells(x,y,z) $\Rightarrow$ Criminal(x)
2. Minor(Jimmy)
3. $\exists$x Owns(Jimmy,x) $\wedge$ Beer(x)
4. $\forall$x Owns(Jimmy,x) $\wedge$ Beer(x) $\Rightarrow$ Sells(Nathan,x,Jimmy)
5. American(Nathan)
6. $\forall$x Beer(x)$\Rightarrow$Alcohol(x)

- Consider all the base terms: Nathan, Jimmy, B1, B2, ….
- Start instantiating #6

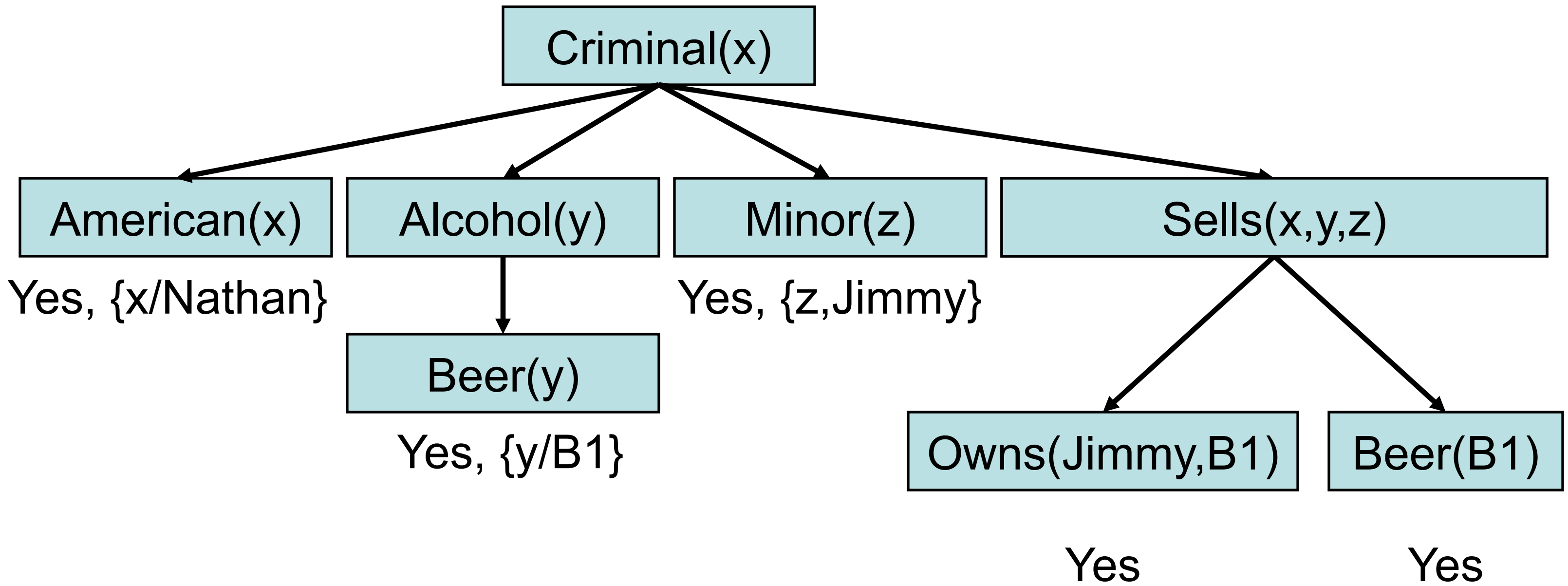
Beer(Nathan) $\Rightarrow$ Alcohol(Nathan)	using {x/Nathan}
Beer(Jimmy) $\Rightarrow$ Alcohol(Jimmy)	using {x/Jimmy}
Beer(B1) $\Rightarrow$ Alcohol(B1)	using {x/B1}
Beer(B2) $\Rightarrow$ Alcohol(B2)	using {x/B2}

Leads to a very disorganized (and full) knowledge base!

Backward Chaining

- Goal-directed
- Starts with a goal state
- Moves backward through implications
- Attempts to construct a set of basic sentences in the KB

Backward Chaining



1. American(x) ∧ Alcohol(y) ∧ Minor(z) ∧ Sells(x,y,z) ⇒ Criminal(x)
2. Minor(Jimmy)
3. Owns(Jimmy,B1)
4. Beer(B1)
5. Owns(Jimmy,x) ∧ Beer(x) ⇒ Sells(Nathan,x,Jimmy)
6. American(Nathan)
7. Beer(x)⇒Alcohol(x)

Failures of Modus Ponens

- We've been using modus ponens as the primary tool for inference
  - But modus ponens does not allow us to deduce new implications, it only derives atomic conclusions

$P(x) \Rightarrow Q(x)$	$Q(x) \Rightarrow S(x)$
$\neg P(x) \Rightarrow R(x)$	$R(x) \Rightarrow S(x)$

Show that $S(A)$ is true

  - Problem is that $\neg P(x) \Rightarrow R(x)$ cannot be converted into Horn form
- We need a more powerful inference rule!
  - Resolution!

Resolution

- Since beta cannot be both true and false, one of the disjuncts must be true

$$\frac{\alpha \vee \beta \ , \neg \beta \vee \gamma}{\alpha \vee \gamma}$$

- Implication is transitive

$$\frac{\alpha \Rightarrow \beta \quad , \quad \beta \Rightarrow \gamma}{\alpha \Rightarrow \gamma}$$

Modus Ponens is a Simplified version of Resolution

- Resolution

$$\frac{\alpha \Rightarrow \beta \quad , \quad \beta \Rightarrow \gamma}{\alpha \Rightarrow \gamma}$$

- Modus Ponens

$$\frac{\alpha, \alpha \Rightarrow \beta}{\beta} \qquad \textit{is the same as} \qquad \frac{True \Rightarrow \alpha, \alpha \Rightarrow \beta}{True \Rightarrow \beta}$$

- Thus, resolution is a more general (and more powerful) format than modus ponens

Resolution has a Canonical Form

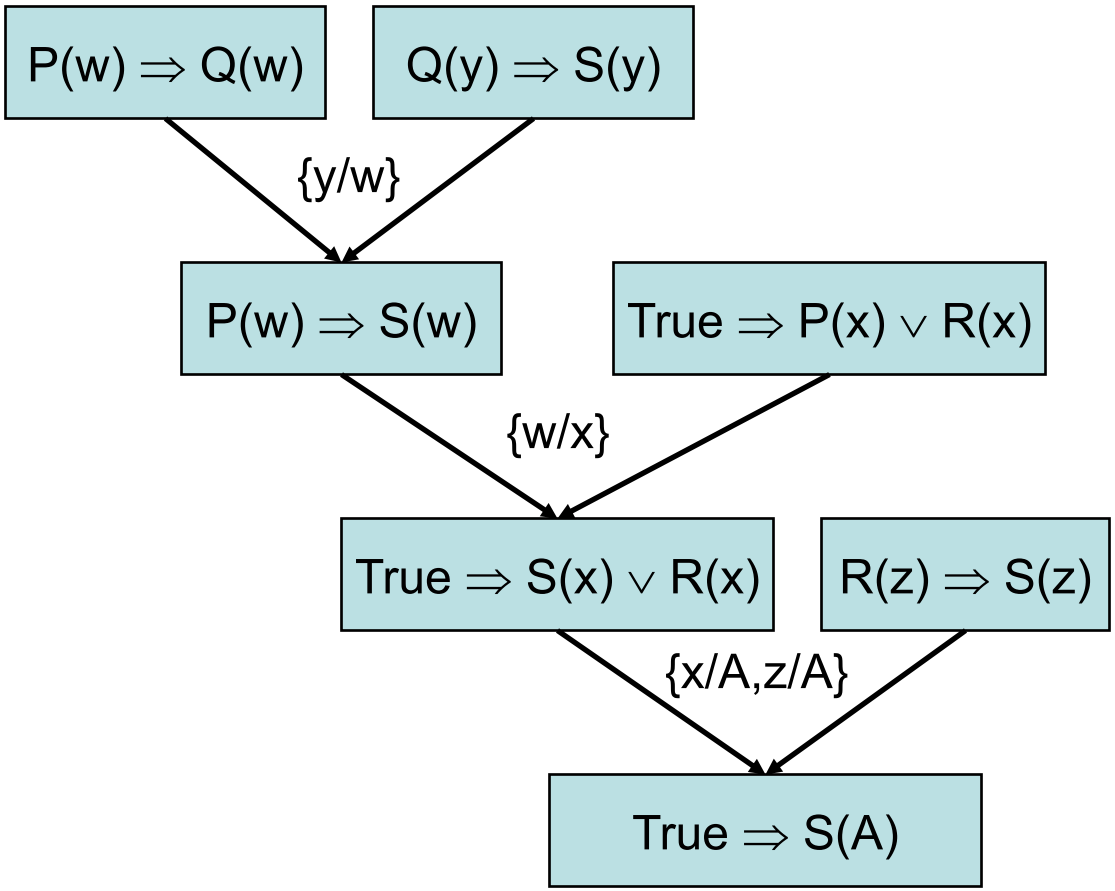
- Using this variant of resolution:

$$\frac{\alpha \vee \beta\ ,\ \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

- We define the conjunctive normal form as the conjunction of all sentences in the knowledge base, where each sentence is a disjunction of literals

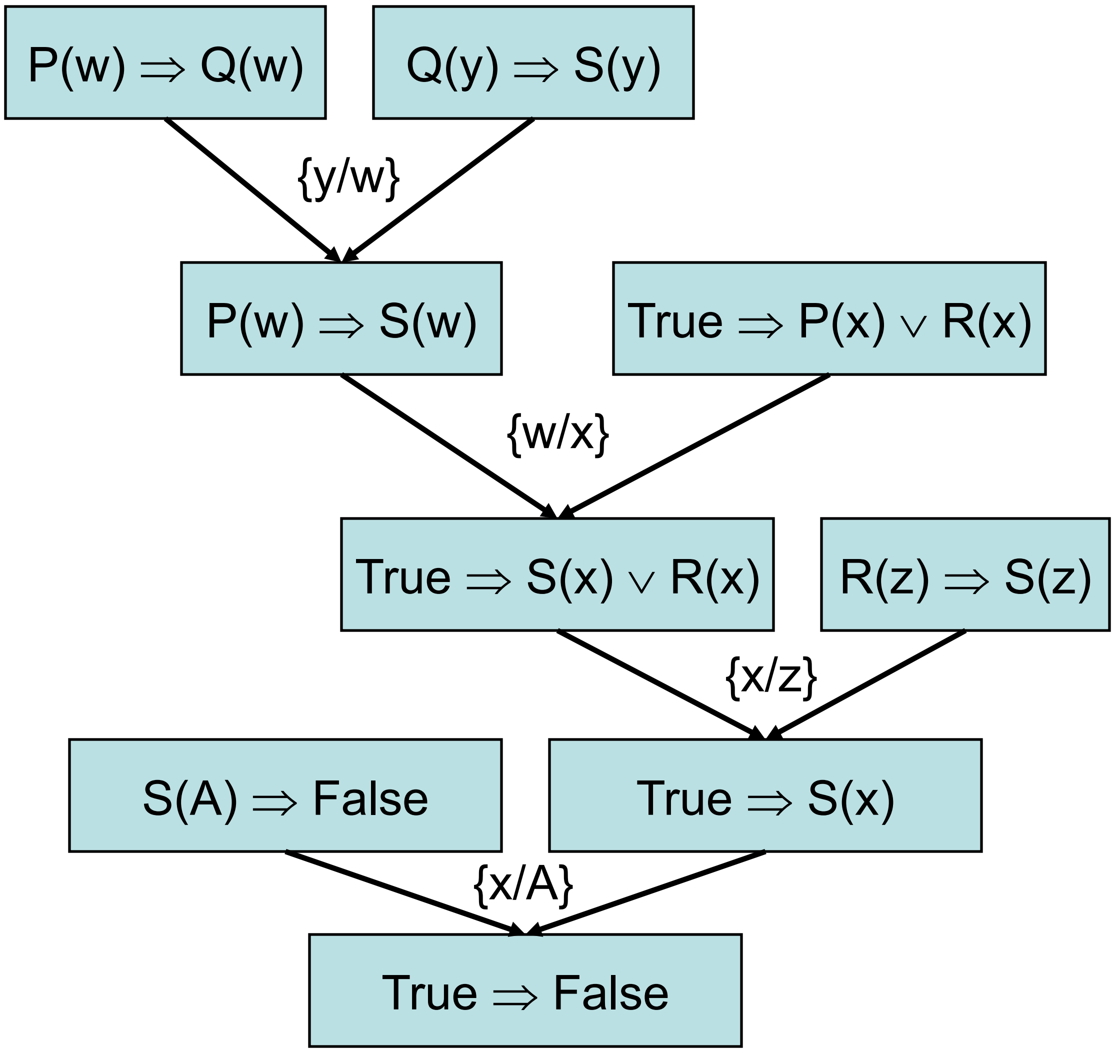
$$(p_1 \vee p_2 \vee ... \vee p_n) \wedge (q_1 \vee q_2 \vee ... \vee q_n) \wedge ...$$

Resolution Proofs



- KB:
  - $P(w) \Rightarrow Q(w)$
  - $Q(y) \Rightarrow S(y)$
  - $\neg P(x) \Rightarrow R(x)$
  - $R(z) \Rightarrow S(z)$
- Could be rewritten in CNF
  - $\neg P(w) \vee Q(w)$
  - $\neg Q(y) \vee S(y)$
  - $P(x) \vee R(x)$
  - $\neg R(z) \vee S(z)$
- Or more simply as:
  - $P(w) \Rightarrow Q(w)$
  - $Q(y) \Rightarrow S(y)$
  - $True \Rightarrow P(x) \vee R(x)$
  - $R(z) \Rightarrow S(z)$
- Prove S(A) follows

Resolution Proofs



- Some statements are valid, but cannot be shown using resolution
  $P \vee \neg P$
- Refutation (proof by contradiction)
  - To prove S(A) is true, assume S(A) is false and show a contradiction

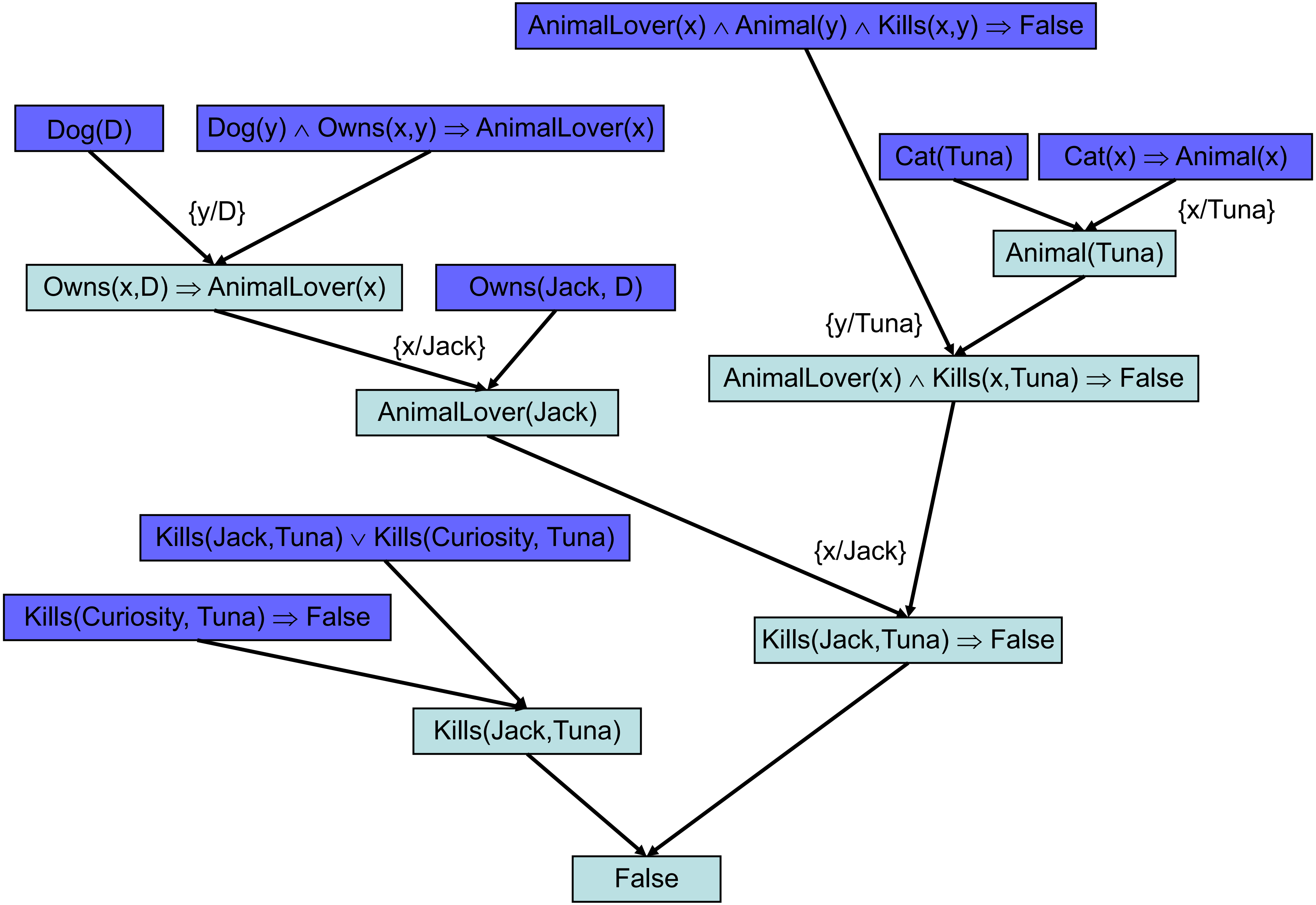
One last Example

- Every dog owner is an animal lover
- No animal lover kills an animal
- Either Jack or Curiosity killed the cat, who is named Tuna
- A cat is an animal
- Jack owns a dog

- Did Curiosity kill the cat?

- $Dog(y) \wedge Owns(x,y) \Rightarrow AnimalLover(x)$
- $AnimalLover(x) \wedge Animal(y) \wedge Kills(x,y) \Rightarrow False$
- $Kills(Jack,Tuna) \vee Kills(Curiosity, Tuna)$
- $Cat(Tuna)$
- $Cat(x) \Rightarrow Animal(x)$
- $Dog(D)$
- $Owns(Jack, D)$

- $Kills(Curiosity, Tuna) \Rightarrow False$



AnimalLover(x) ∧ Animal(y) ∧ Kills(x,y) ⇒ False
Dog(D)
Dog(y) ∧ Owns(x,y) ⇒ AnimalLover(x)
Cat(Tuna)
Cat(x) ⇒ Animal(x)
{y/D}
{x/Tuna}
Animal(Tuna)
Owns(x,D) ⇒ AnimalLover(x)
Owns(Jack, D)
{y/Tuna}
{x/Jack}
AnimalLover(x) ∧ Kills(x,Tuna) ⇒ False
AnimalLover(Jack)
Kills(Jack,Tuna) ∨ Kills(Curiosity, Tuna)
{x/Jack}
Kills(Curiosity, Tuna) ⇒ False
Kills(Jack,Tuna) ⇒ False
Kills(Jack,Tuna)
False

Coming Up

- Friday: Practical uses for Logical Inference Systems
- Problem Set #3 out today!
- Special Guest Lecture on Monday!