Reasoning under Uncertainty

CPSC 470 – Artificial Intelligence

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STRIPS planner

STanford Research Institute Problem Solver

- "Holy Roman Empire" naming
- Represent states and goals in first-order logic

  At(Home) ∧ Have(Milk) ∧ Have(Drill) ∧ Have(Banana)

- Assume existential quantification of variables

  At(x) ∧ Sells(x, Milk)

Why STRIPS is Insufficient for many Domains

- Hierarchical plans
  - Allow for more complex plans by varying level of abstraction
- Resource Limitations
  - Consumption and generation of resources
  - Time as a resource
    - Based on situation calculus, assumes all actions take place simultaneously and in one unit of time
    - Actions in a plan may have durations, deadlines, and time windows
- Complex conditions
  - No conditionals in STRIPS
  - No universals in STRIPS
- Dealing with incomplete or inaccurate information
  - Conditional planning
  - Execution monitoring

Failures of Logic

- First-order logic represents a certainty
  - $\forall p$ Symptom(p,Toothache) $\Rightarrow$ Disease(p,Cavity)
- To make the rule true, we must add an almost unlimited set of causes
  - $\forall p$ Symptom(p,Toothache) $\Rightarrow$ Disease(p,Cavity) $\vee$ Disease(p, GumDisease) $\vee$ Disease(p, SinusInfection) $\vee$ …
- Conversion to a causal rule does not help
  - $\forall p$ Disease(p,Cavity) $\Rightarrow$ Symptom(p,Toothache)

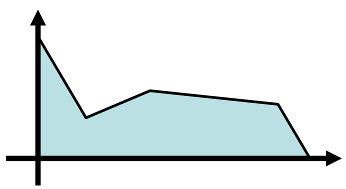
Why does Logic Fail?

- Laziness
  - Too much work to list entire sets of consequents or antecedents
  - List all possible causes for a toothache
- Theoretical Ignorance
  - No complete theory for the domain exists
  - Describe precisely the conditions that cause cancer
- Practical Ignorance
  - Even if we know all the rules, we may be uncertain about a particular event
  - What was the white blood cell count of the patient two years ago?

Probability provides a way of summarizing the uncertainty that comes from our laziness and ignorance

Basic Probability

- Assume a basic understanding of probability theory, but as a quick review:
- Unconditional (prior) probability:
  - P(Cavity) = 0.1
- Random Variables
  - P(*Weather* = snow) = 0.05
- Probability distribution



- Conditional (posterior) probability:
  - P(Cavity|Toothache) = 0.8

Basic Probability II

- Axioms

$0 \leq P(A) \leq 1$

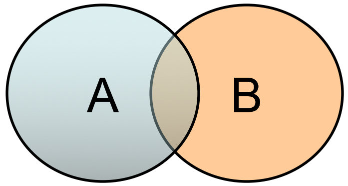
$P(True) = 1$

$P(False) = 0$

$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$

$P(\neg A) = 1 - P(A)$

$P(A \wedge B) = P(A|B)\,P(B)$ (the product rule)



Bayes' Rule

- From the product rule

$$P(A \wedge B) = P(A|B)\,P(B)$$

$$P(A \wedge B) = P(B|A)\,P(A)$$

- Bayes' Rule:

$$P(A|B)\,P(B) = P(B|A)\,P(A)$$

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$$

Application of Bayes' Rule

- Medical example:
  - 1 in 20 patients reports a stiff neck
  - 1 in 50,000 patients has meningitis
  - Meningitis causes a stiff neck 50% of the time
  - If I have a stiff neck, what is the chance that I have meningitis?

- Apply Bayes' Rule:

$P(S) = 1/20$

$P(M) = 1/50000$

$P(S|M) = 0.50$

$$P(M|S) = \frac{P(S|M)P(M)}{P(S)}$$

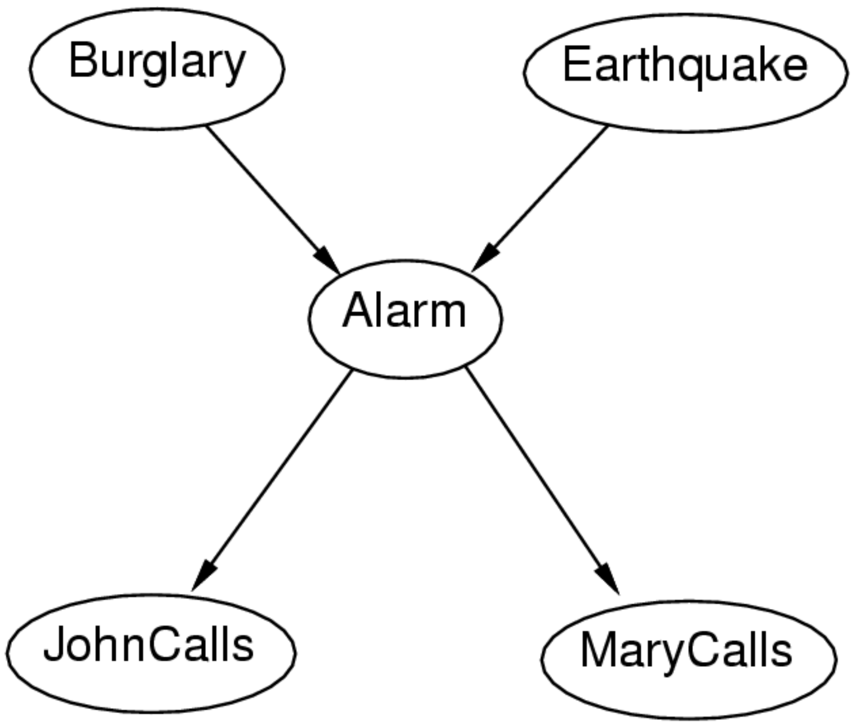
$$P(M|S) = \frac{0.5 \times 1/50000}{1/20}$$

$P(M|S) = 0.0002$

Uncertainty and Rational Decisions

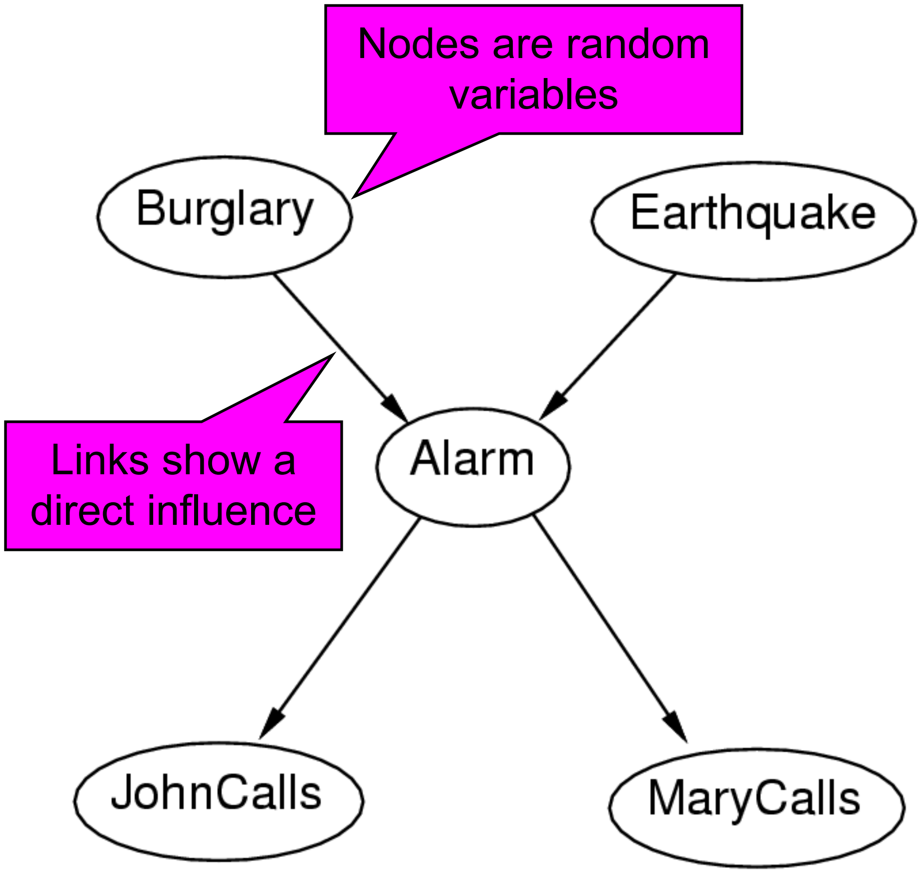
- Consider these plans:
  - Plan #1: Pay $10 for a 75% chance of event X
  - Plan #2: Pay $30 for a 10% chance of event X
  - Plan #3: Pay $90 for a 79% chance of event X
- Which plan should you choose?
  - If the event X is “receive 2 bonus points on PS5”
  - If the event X is “surviving an operation”
- An agent must have a set of preferences in order to make decisions in an uncertain world

Probabilistic Reasoning Systems



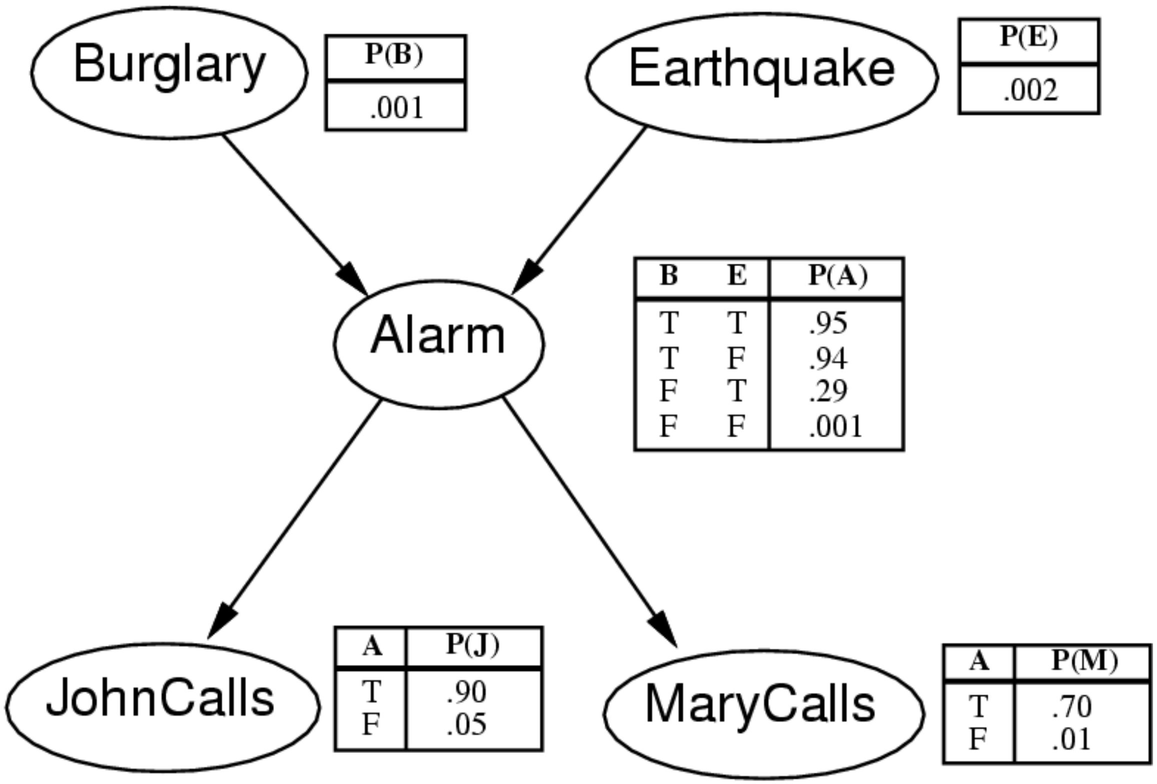
- We've seen the syntax and semantics of probability
- Now we look at an inference mechanism:
  Belief networks

A Basic Belief Network



- You have a new home alarm that responds
  - Accurately to burglaries
  - Occasionally responds to earthquakes
- When the alarm rings, your neighbors call you at work
  - John always calls, but sometimes confuses the telephone for the alarm
  - Mary sometimes misses the alarm, but only calls when the alarm actually rings

A Basic Belief Network



P(B)
.001

P(E)
.002

B	E	P(A)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J)
T	.90
F	.05

A	P(M)
T	.70
F	.01

A conditional probability table gives the likelihood of a particular combination of values

- You have a new home alarm that responds
  - Accurately to burglaries
  - Occasionally responds to earthquakes
- When the alarm rings, your neighbors call you at work
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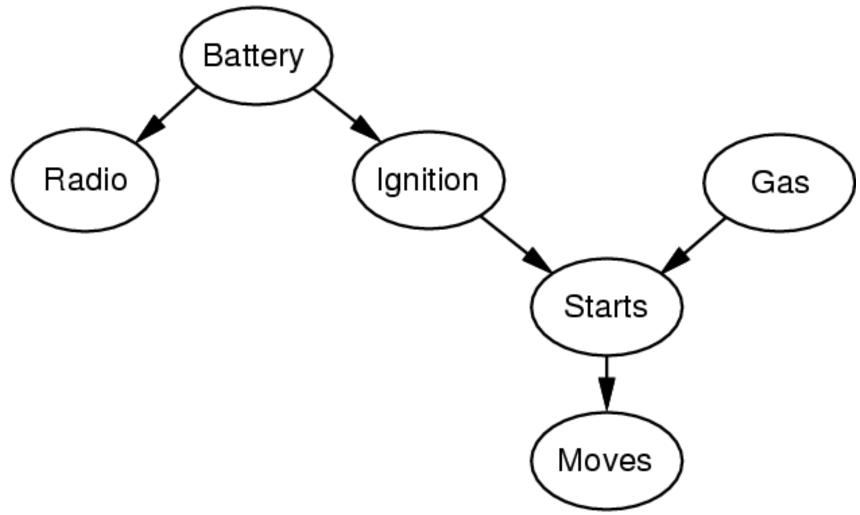
Conditional Independence Relations in Belief Nets



- Each variable is conditionally independent of its non-descendants, given its parents
- Presence of Gas and a working Radio are
  - Independent given evidence about the ignition
  - Independent given evidence about the battery
  - Independent given no evidence
  - Dependent given evidence that the car starts
    - If the car does not start, but the radio plays, then the chance of being out of gas is increased

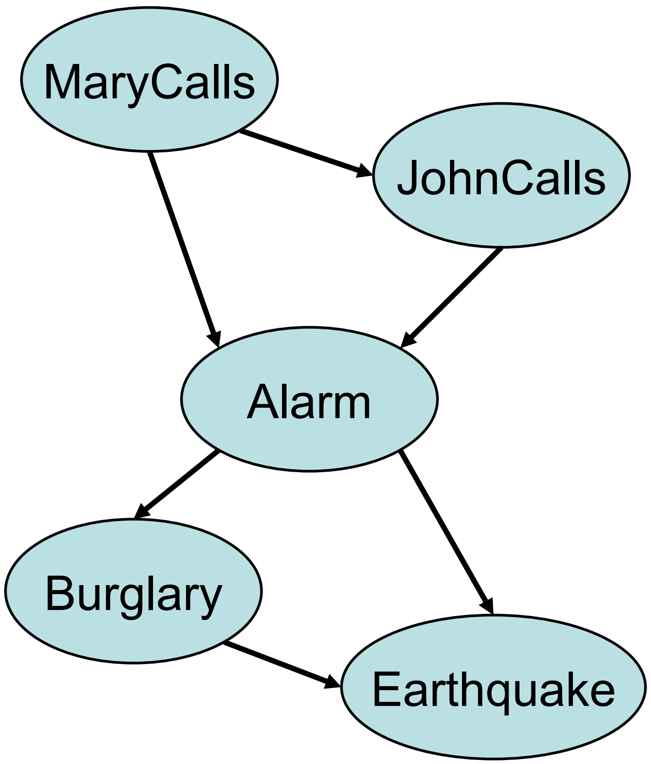
Incremental Construction of Belief Nets

- Conditional Independence Property

  P(X | A, B, C…) = P(X | Parents(X))

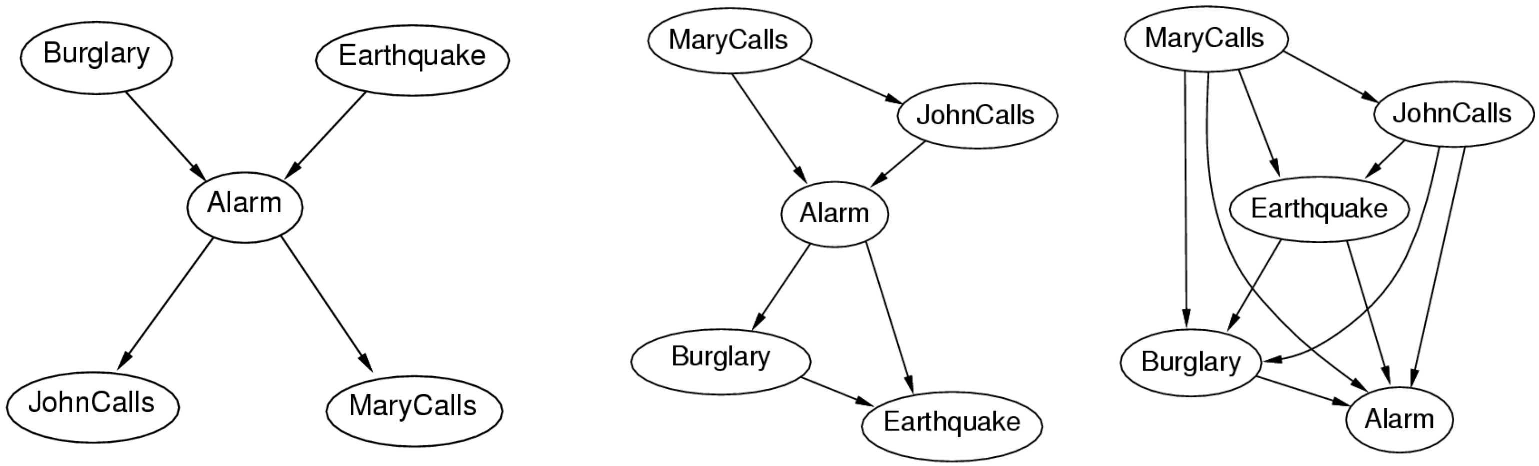
  where Parents(X) gives those nodes (A, B, etc.) that are the parent nodes of X
- Incremental belief net construction:
  1. Choose the set of relevant variables
  2. Choose an ordering for the variables
  3. While there are variables left:
     a. Pick a variable ***a1*** and add a node to the network for it.
     b. Set ***Parents(a1)*** to the minimal set of nodes that satisfies the conditional independence property
     c. Define the conditional probability table for node ***a1***

Incremental Construction of Belief Networks



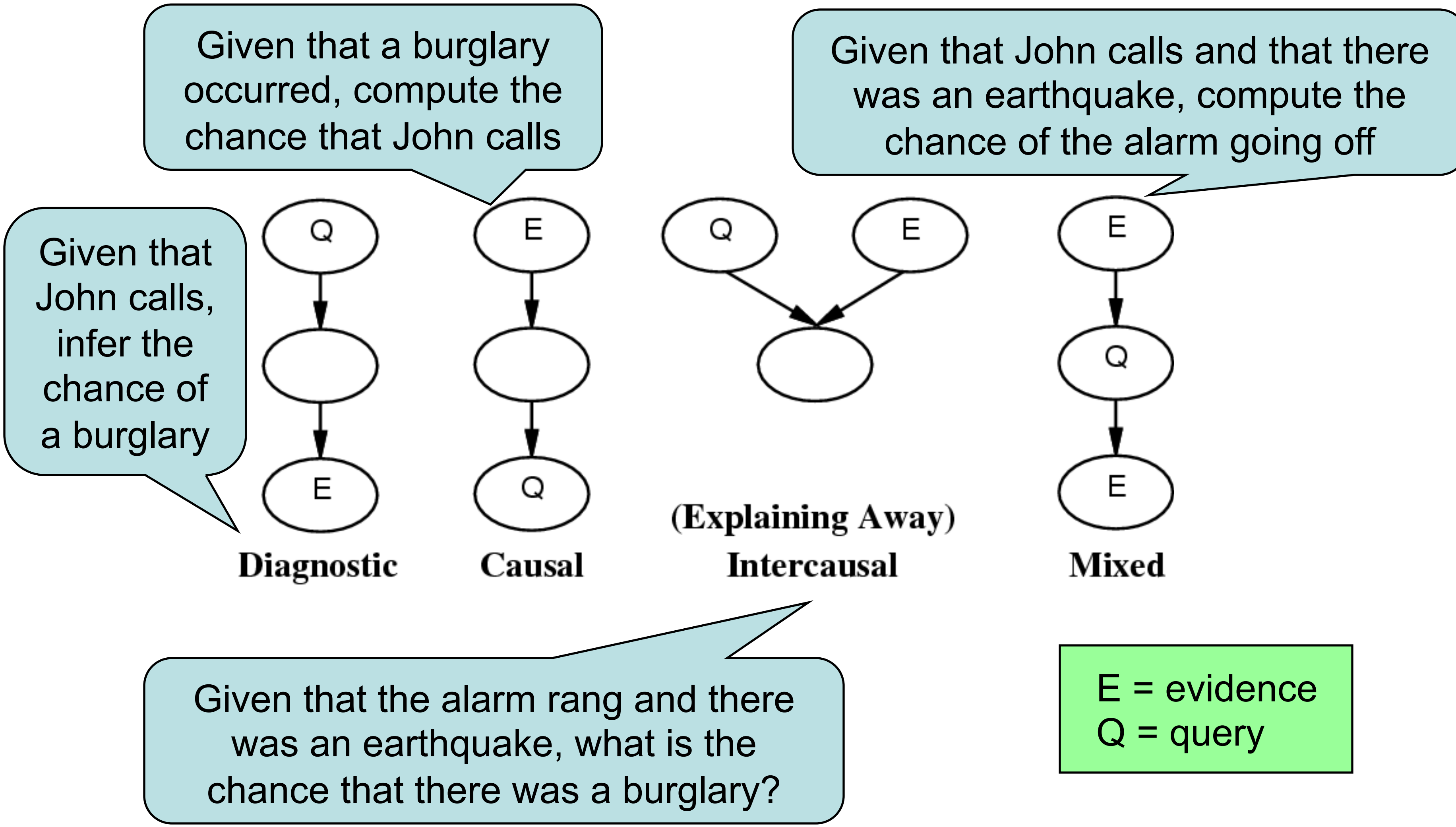
- Add *MaryCalls*
- Add *JohnCalls*
  - Dependence with *MaryCalls* since P(John|Mary)≠P(John)
- Add Alarm
  - More likely if both calls are made
- Add Burglary
  - Phone calls don't tell us anything about the chance of a burglar, but the alarm does
- Add Earthquake
  - Alarm acts as earthquake predictor
  - Presence of a burglar helps determine whether or not an earthquake occurred

Incremental Construction of Belief Networks

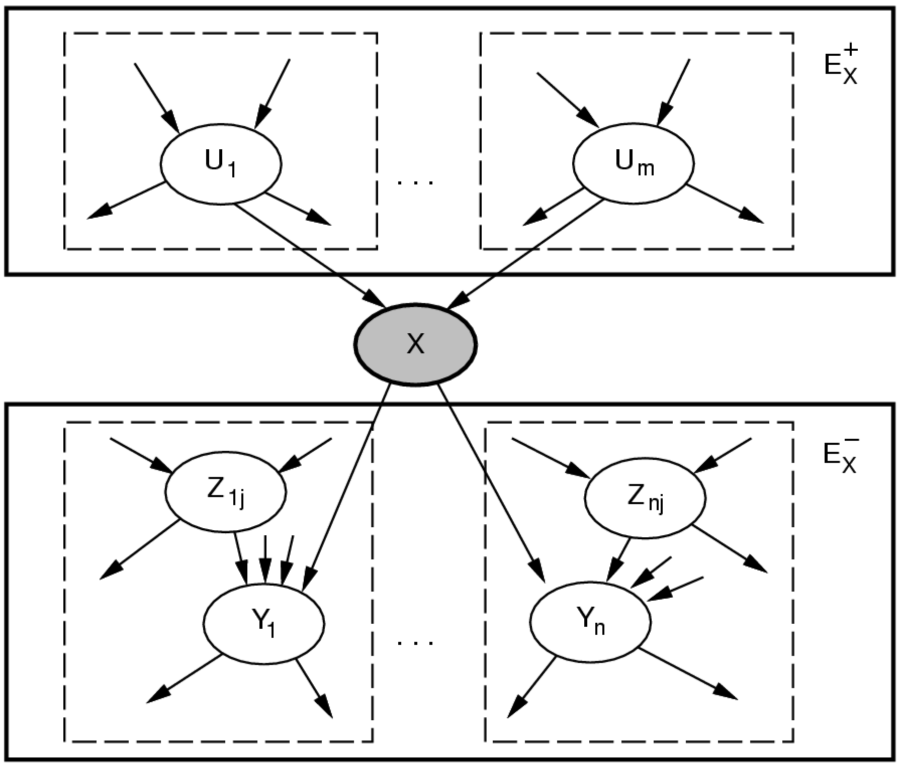


- Order in which you add nodes can make a difference on the number of links
- "Correct order" to add nodes is to add the "root causes" first and then the variables they influence, and so on…
- If we stick to a causal model, we need fewer probabilities and these probabilities will be easier to create

Types of Inference



Inference with Belief Nets

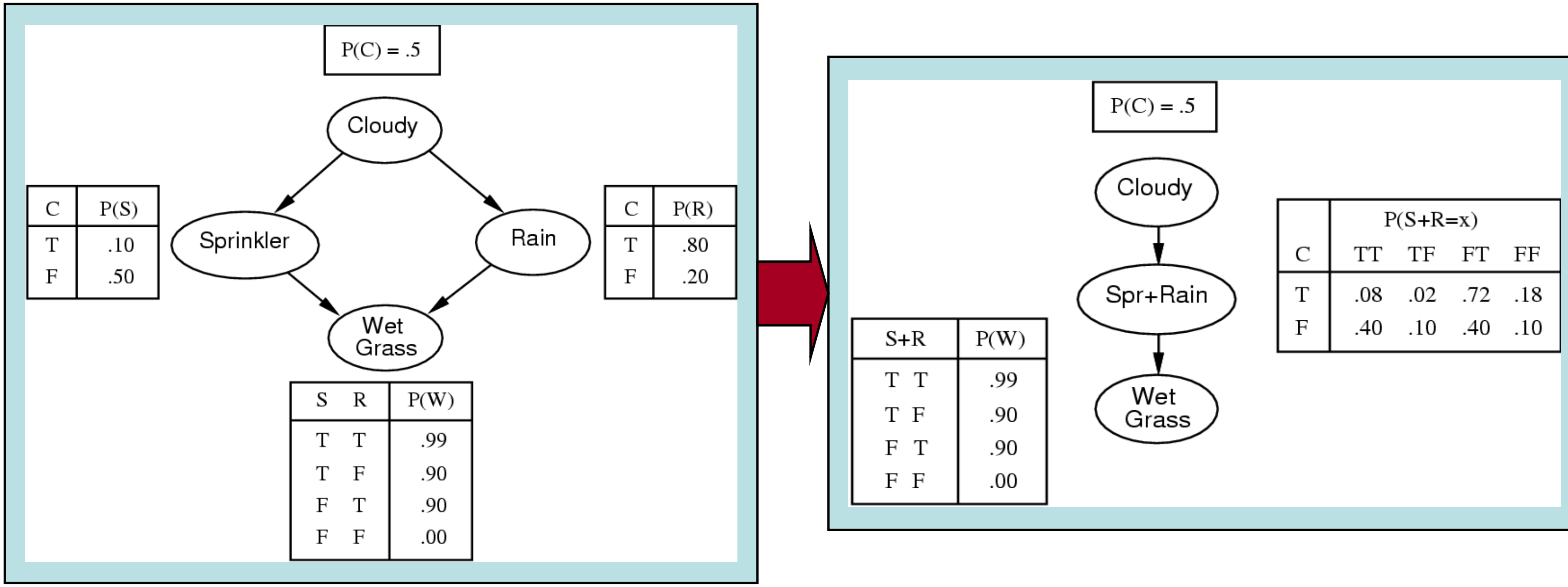


- Singly connected graph
  - only one path between any two nodes
- Inference in singly-connected graphs
  - Causal support for X:
    - Evidence "above" X (from the direction of X's parents)
  - Evidential support for X:
    - Evidence "below" X (from the direction of X's children)
- Multiply connected graph
  - multiple paths between nodes

Three Techniques for Inference in Multiply Connected Belief Networks

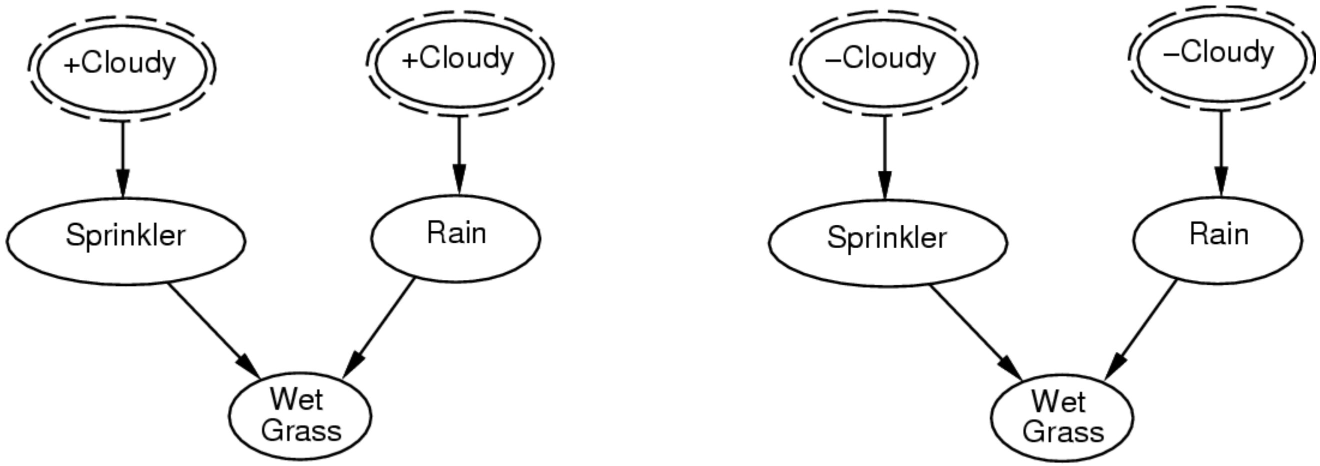
- Clustering
  - Transform the network by merging nodes
    - Probabilistically equivalent
    - Topologically different
- Cutset Conditioning
  - Transform by instantiating variables to values and re-evaluating the network
- Stochastic Simulation
  - Generate many consistent concrete models and approximate an exact evaluation

Clustering in Belief Networks



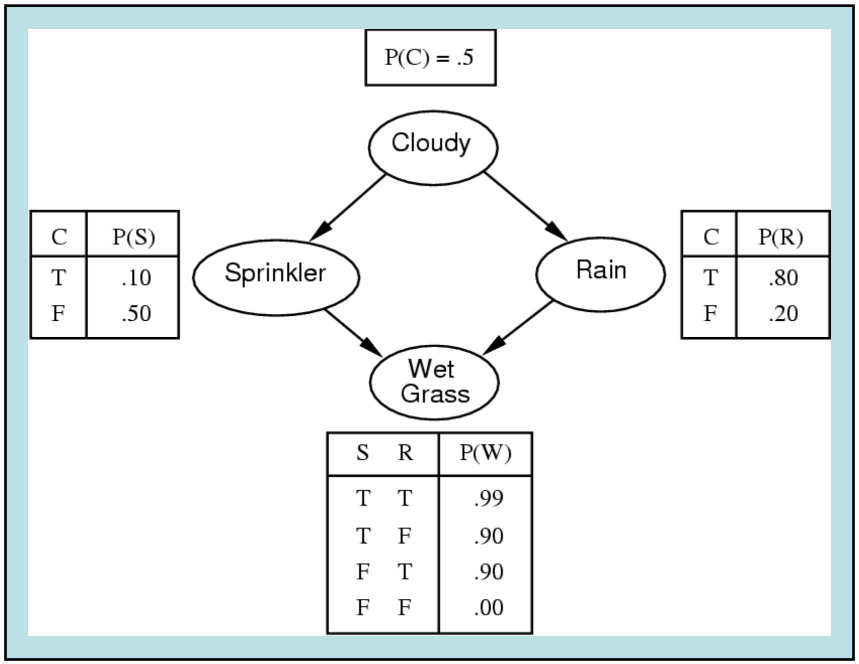
- Try to change a multiply-connected graph into a singly-connected graph by merging nodes
- Introduces complexity into each merged node, but the payoff is in the use of simpler inference mechanisms

Cutset Conditioning



- “Opposite” of clustering: transforms the network into several simpler graphs
  - Number of graphs is exponential in the size of the cutset
- Transform by instantiating variables to values and re-evaluating the network

Stochastic Simulation Methods



- If we want to estimate P(WetGrass | Rain) then we just start at a root node and simulate a large number of trials
  - Choose a value for Cloudy based on the probability
  - Propagate this value through the network
  - Count the number of instances of WetGrass and Rain to estimate the probability

Where do Probabilities come from?

- Frequentist
  - Probabilities come from experiments
  - “9 out of 10 dentists agree”
- Objectivist
  - Probabilities are real aspects of the universe
  - Propensities of objects to act in certain ways
- Subjectivist
  - Probabilities characterize an agent’s beliefs
  - “In my opinion, there is a 30% chance of success”

Computing Probabilities and Reference Classes

- Probability that the sun will exist tomorrow

Undefined	There has never been an experiment that tested the existence of the sun tomorrow
1	In all previous experiments (previous days), the sun has continued to exist
$1-\varepsilon$	Where ε is the proportion of stars in the universe that go nova every day
$\frac{(d+1)}{(d+2)}$	Where d is the number of days that the sun has existed so far (formula due to Laplace)
???	Can be derived from the type, age, size, and temperature of the sun

Administrivia

- Coming up next…learning!