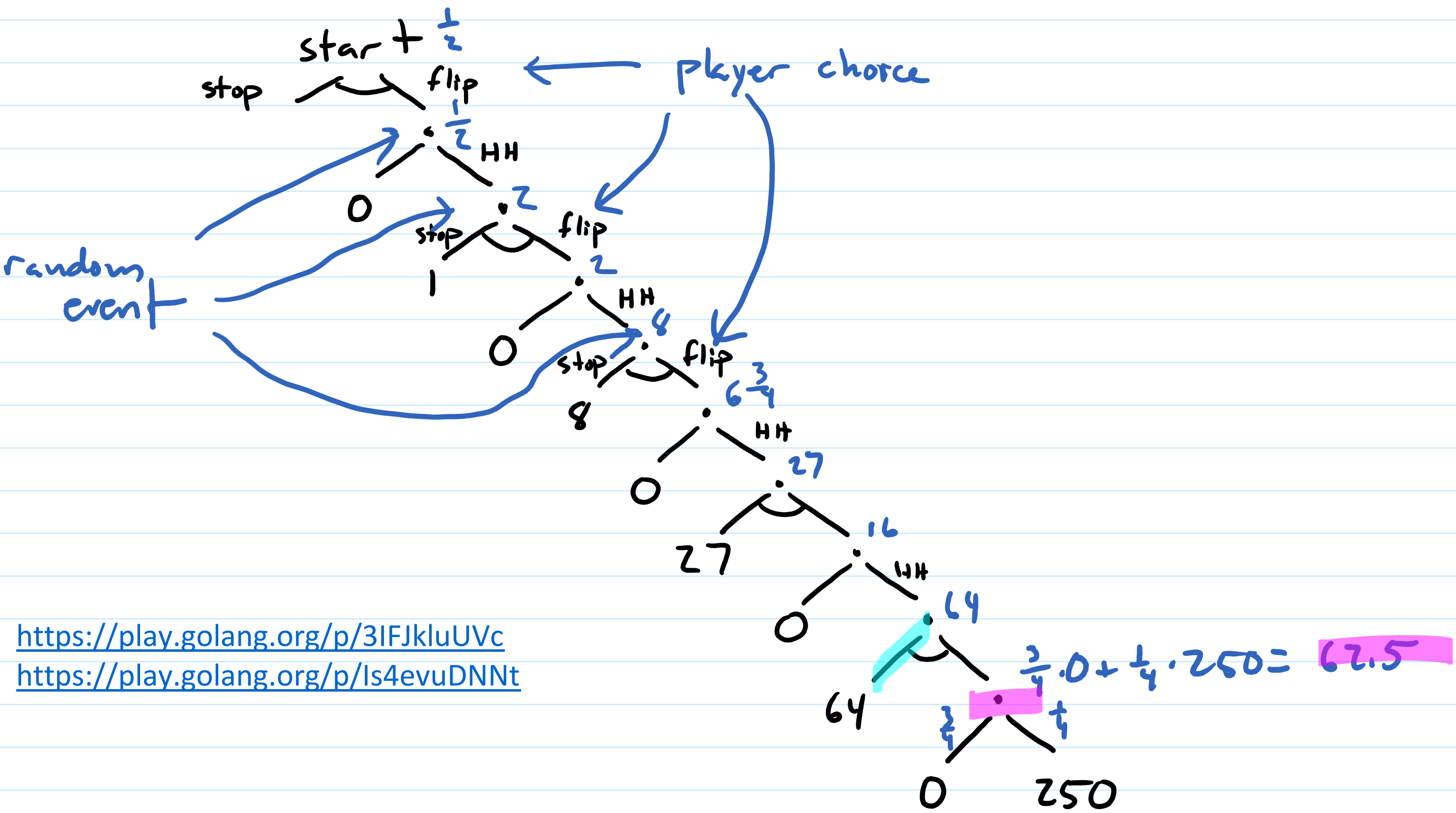


https://play.golang.org/p/3IFJkluUVc
https://play.golang.org/p/Is4evuDNNt

Analysis of 1-player Finite Probabilistic Games

$E[pos]$ = expected winnings having reached position pos

For final positions pos, $E[pos]$ determined by rules of game

For non-final choice positions

$$E[pos] = \max_{\text{choice } c} E[\underline{next(pos, c)}]$$

position resulting from choice c in position pos

For non-final random event positions

$$E[pos] = \sum_{\text{outcome } \sigma} P(\sigma) \cdot E[next(pos, \sigma)]$$

```
for every final position pos
    E[pos] ← payout(pos)

for every non-final position pos in reverse order of topological sort
                                   (suffices to be in order of ↑
                                    turns from end)
    if pos is a choice position
        max ← -∞
        argmax ← NIL
        for every choice c
            e ← E[next(pos,c)]
            if e > max
                e ← max
                argmax ← c
        E[pos] ← e
        OPT[pos] ← argmax
    else
        e ← 0.0
        for every outcome σ
            e ← e + P(σ) · E[next(pos,σ)]
        E[pos] ← e
```

Coins: Start with n coins.
On each turn, flip as many of your remaining coins as you wish.
If #T ≥ #H, lose all the T
Else earn #H points

On each turn, flip as many of your remaining coins as you wish.
If $\#T \geq \#H$, lose all the T
Else earn $\#H$ points
Win at X points
Lose if no coins left and $< X$ points

Yahtzee

anchor: position at start of turn

component: set of positions reachable from each anchor

number of anchors:

2	ones	: blank, 0, ..., 5	7
2	twos	: blank, 0, 2, 4, ..., 10	7
⋮		⋮	
2	sixes	:	7
2	3K	: blank, 0, 5, ..., 30	28
2	4K	:	28
2	C	:	28
2	LS	: blank, 0, 40	3
2	SS	: blank, 0, 30	3
2	FH	: blank, 0, 25	3
3	Y	: blank, 0, 50, ..., 1250	15
64	upper total		

$\approx$ 1 trillion

$\frac{3}{4}$ million anchors

$\approx$ 1600 pos / anchor

$\approx$ 1.2 billion pos total

252+462+252+462+252

$\approx$ 1600 combos of dice / rerolls

1.6 quadrillion pos

modification: E[pos] = expected future score from pos

For non-final choice positions

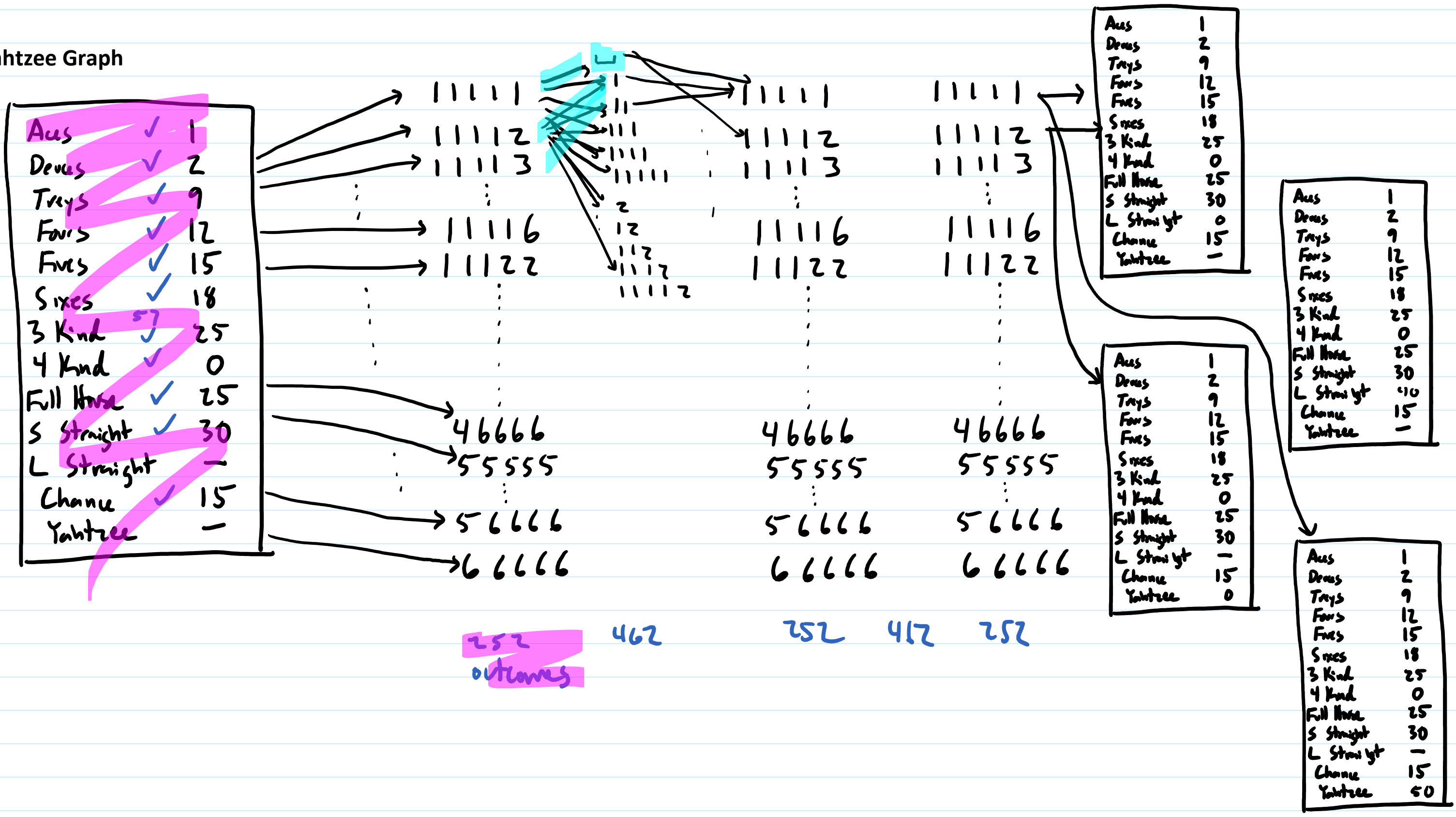
$$E[pos] = \max_{\text{choice } c} E[next(pos, c)] + \underbrace{score(pos, c)}_{\text{reward for choice } c \text{ in pos}}$$

For non-final random event positions

$$E[pos] = \sum_{\text{outcome } \sigma} P(\sigma) \cdot \Big(E[next(pos, \sigma)] + score(pos, \sigma) \Big)$$

anchors =

Yahtzee Graph



Two-player Zero-sum, probabilistic finite games

$E[pos]$ = expected wins for P1 from pos
($= P(\text{P1 wins} \mid \text{game @ pos})$)
if no draws

For P1 choice position

$$E[pos] = \max E[next(pos, c)]$$

For P2 choice position

$$E[pos] = \cancel{\max}\ \min\ E[next(pos, c)]$$

For non-final random event positions

$$E[pos] = \sum_{\substack{\text{outcome} \\ \sigma}} P(\sigma) \cdot E[next(pos, \sigma)]$$

2-player Yahtzee anchors:

Y : 3^2
other cats : 2^{24}
upper sub : 64^2
score diff : $\underline{3000}$ (−1500 … 1500)

$\approx$ 100 trillion anchors

$\approx$ 100 billion sec @ 1000 anchors / sec

$\approx$ 3000 years

Aces	1	1	0	1
Deuces	2	2	0	2
Treys	9	9	0	9
Fours	12	12	0	12
Fives	15	15	0	15
Sixes	18	18	0	18
3 Kind	25	25	0	25
4 Kind	0	0	0	0
Full House	25	25	0	25
S Straight	30	30	0	30
L Straight	—	—	—	—
Chance	15	15	5	15
Yahtzee	—	—	—	—

2-player Yahtzee variant:

1) get score distribution of optimal solitaire player

2) compute strategy that maximizes the probability of beating the optimal solitaire player