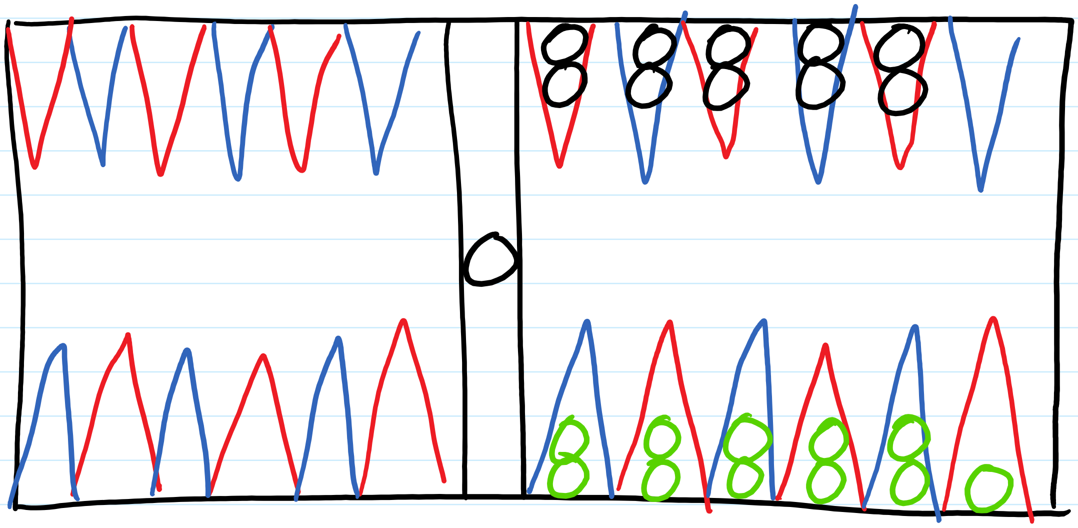
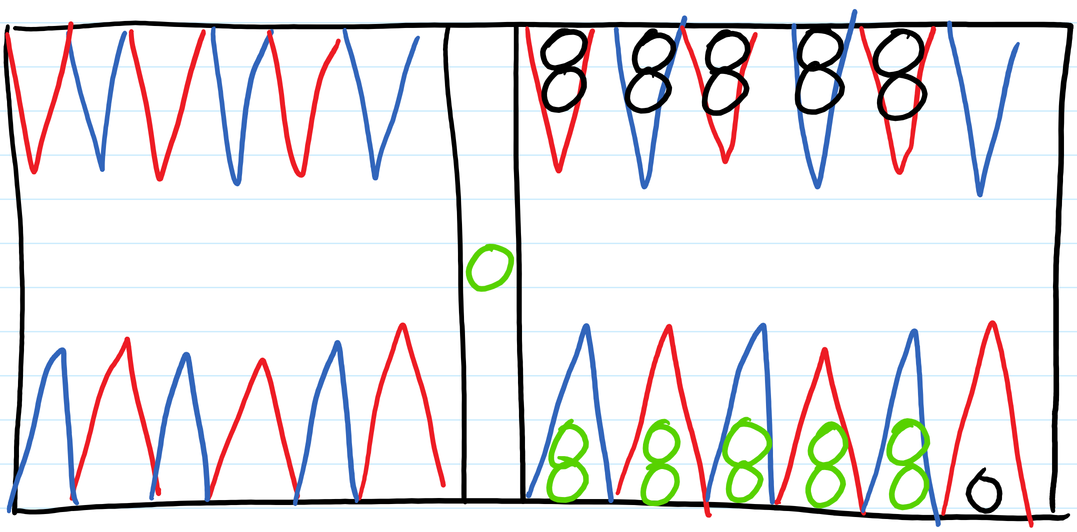
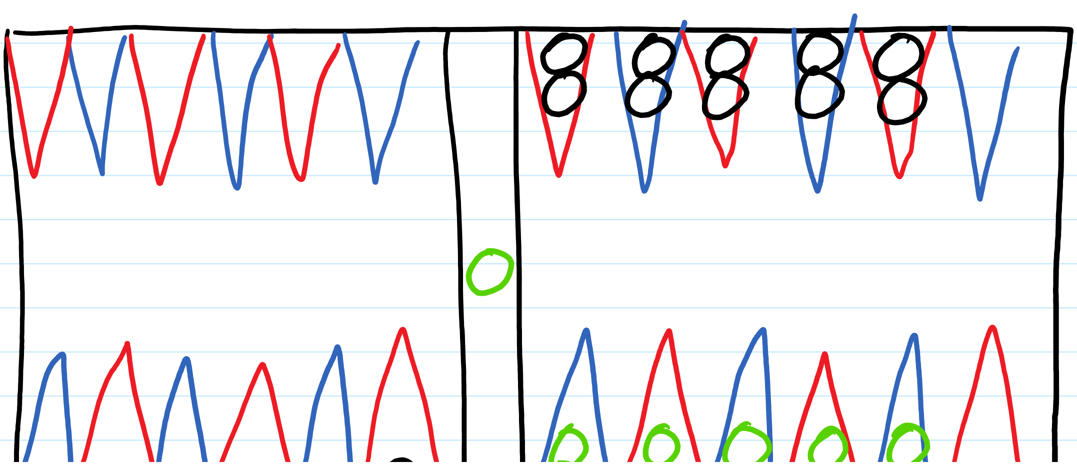
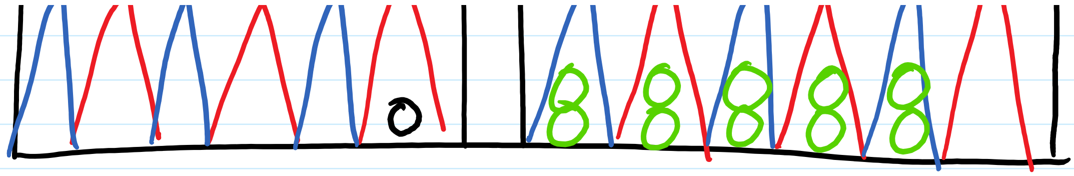
Backgammon



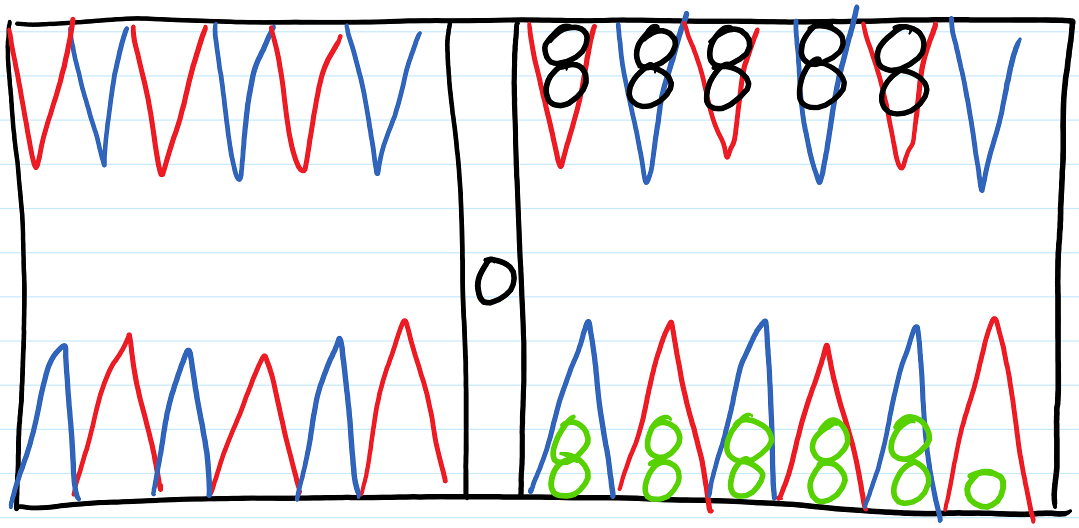
Black rolls 1-6



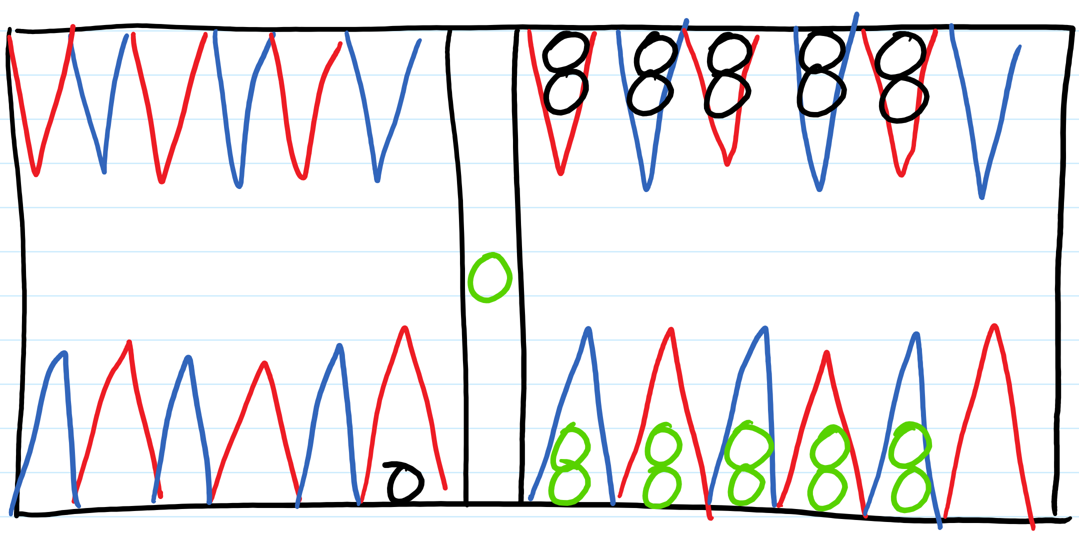




Green rolls 5-6 4-4 3-6 1-6
Black rolls 1-2 6-3 3-2



Black rolls 6-5 4-3 5-5 2-4 1-6
Green rolls 2-3 1-3 1-2 3-2



same position as many turns ago – long cycles (many pos on same cycle)

3+ Player Pig

next / next next / next next next = prev

$E_1[x, y, z]$ = expected wins for next player when score is x-y-z

$E_{1xyz}[t]$ = expected wins for next when score is x-y-z; subtotal t

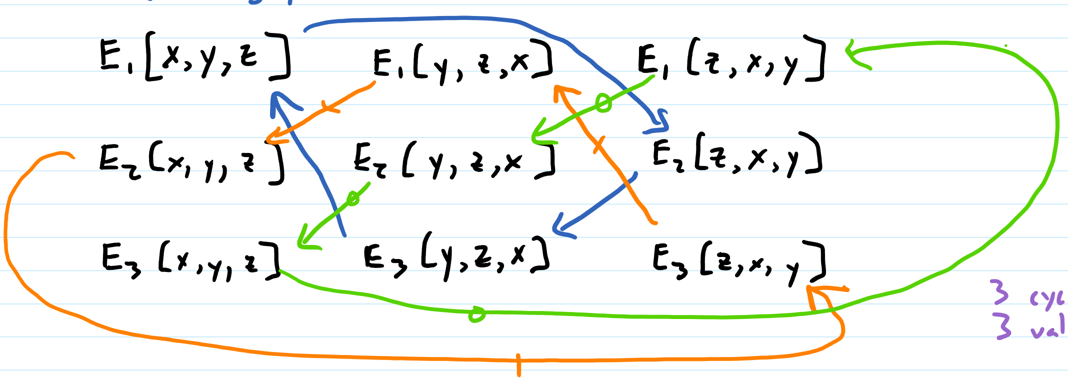
$$= \begin{cases} 1.0 & \text{if } x+t \geq T \\ \max\left(E_3[y, z, x+t], \frac{1}{6}\left(E_3[y, z, x] + \sum_{r=2}^{6} E_{1xyz}[t+r]\right)\right) \end{cases}$$

$E_2(x, y, z]$ = expected wins for next next

$$= \sum_{\substack{\text{outcome } o \\ \text{chosen by next} \\ \text{(assuming optimal)}}} P(o) \cdot E_1[y, z, \underbrace{x'}_{\text{score for next after outcome } o}]$$

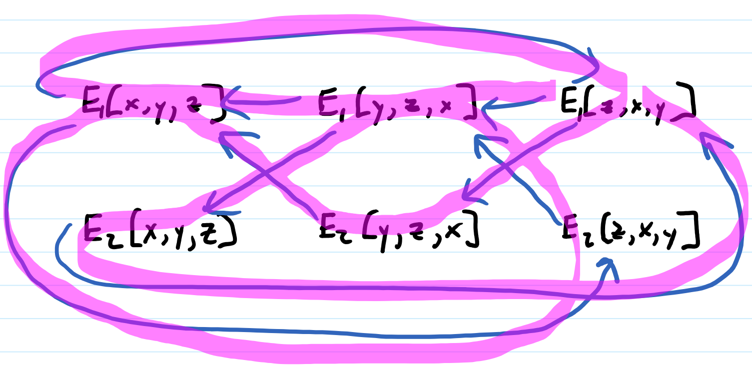
$E_3[x, y, z]$ = expected wins for prev

$$= \sum_{\substack{\text{outcome } o \\ \text{chosen by next} \\ \text{(assuming optimal)}}} P(o) \cdot E_2[y, z, \underbrace{x'}_{\text{score for next after outcome } o}]$$



3 cycles, 3 values each

note $E_1[x,y,z] + E_2[x,y,z] + E_3[x,y,z]$



1 cycle, 6 values

← less memory

which converges faster?

Me Alice Bob

← I hate this guy (does not apply to actual Bobs I know)

$\max\left(\alpha E_3[y, z, x+t], \frac{1}{6}\left(\alpha E_3[y, z, x] + \sum_{r=2}^{6} \alpha E_{1xyz}[t+r]\right)\right)$

So I may value a win by Alice a little

I hate this guy (does not apply to actual Bobs I know)

$$\max\left(\alpha \underbrace{E_3[y,z,x+t]}_{+\,(1-\alpha)E_1[y,z,x+t]},\ \frac{1}{6}\left(\alpha E_3[y,z,x] + (1-\alpha)E_1[y,z,x] + \sum_{r=2}^{6} \alpha E_{1xyz}[t+r] + (1-\alpha)E_{2xyz}[t+r]\right)\right)$$

So I may value a win by Alice a little

\ Simultaneous Play Games

I \ II	R	P	S
Rock	0, 0	−1, 1	1, −1
Paper	1, −1	0, 0	−1, 1
Scissors	−1, 1	1, −1	0, 0

$$A_{\mathrm{I}} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \quad B_{\mathrm{II}} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

zero-sum game: $a_{ij} + b_{ij} = 0$ for all i, j

constant-sum game: $a_{ij} + b_{ij} = C$ for all i, j

constant-sum reduces to zero-sum

1) play zero-sum game $A - C$
2) award I c units

no strategy here, so optimizing 1+2 = optimizing 1

Penalty Kick

	L	R
L	$\frac{1}{2}, -\frac{1}{2}$	1, −1
R	1, −1	$\frac{2}{3}, -\frac{2}{3}$

	W	X	Y	Z
A	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$
B	−1	$\frac{1}{3}$	$\frac{1}{4}$	$-\frac{1}{2}$
C	$\frac{1}{2}$	0	−1	$\frac{1}{2}$

Stag Hunt

	S	H
Stag	2, 2	0, 1
Hare	1, 0	1, 1

non-constant sum

$$\begin{array}{cc} & \text{defense} \\ & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \text{offense} \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \end{array}$$

expected wins for offense when choosing plays 0, 0

values given in pickle/csv

$$a_{ij} = \sum_{w=1}^{5} \underbrace{P(\text{window } w)}_{\text{plays.prob}} \cdot E\left[\underbrace{\text{next}(pos, w)}_{\substack{\text{next position} \\ \text{after pos} \\ \text{given outcome} \\ \text{in window } w}}\right]$$

1) look up outcome in plays.plays[i][j][w]
2) compute next position
3) look up value of next position in pickle/csv

S