CPSC 474/574 - Spring 2018 - Exam #1

Write your name and NetID on this cover page and only on this cover page. Please indicate clearly your final answers to each question.

Problem 1 (10 points): Consider a misere form (last move loses) of Nim in which players may take 1, 2, or 5 stones. Compute the outcome classes for positions with n stones for $0 \leq n \leq 9$.

Problem 2 (10 points): Welter's game is a game played with m coins on a 1-D board with spaces marked $0, 1, ..., k$, each of which can be empty or can hold one coin. Players alternate turns; on each turn, the current player may move any coin to an unoccupied space strictly closer to the 0 end of the board. The game ends when the coins are in spaces $0, ..., m-1$, and the player who made the last move wins.

Compute the Grundy number of each position given below. (the o's represent the coins and the dots represent empty spaces; the rightmost position is position 0).

...oo

..o.o

..oo.

.o..o

.o.o.

.oo..

o...o

o..o.

o.o..

Problem 3 (10 points): Find a winning move for the Kayles position shown below if such a move exists; otherwise simply state that no such move exists. (Grundy numbers for some initial Kayles positions are given below for reference.)

xxxxxx xxxxx x

Kayles Position		x	xx	xxx	xxxx	xxxxx	xxxxxx
Grundy Number	0	1	2	3	1	4	3

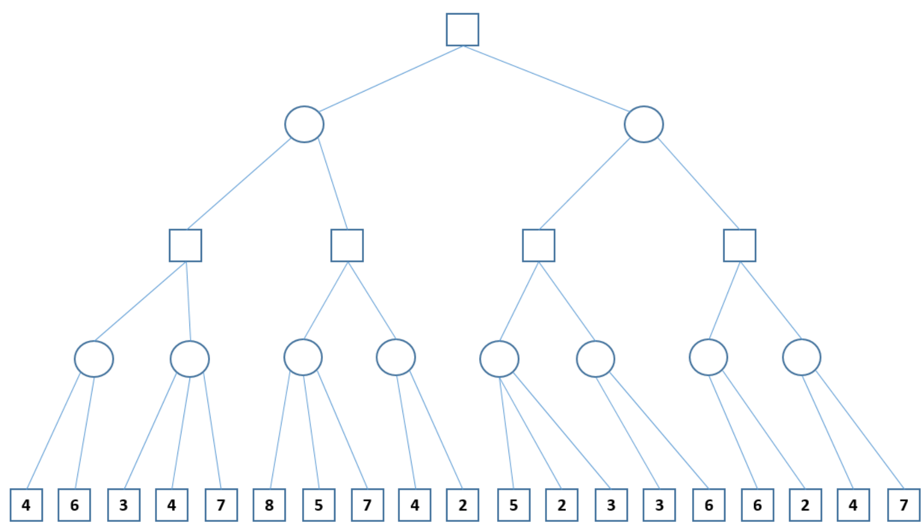
Problem 4 (10 points): Consider a solitaire version of Welter's game. On each turn, the player selects a movable coin, and the coin is randomly placed on an unoccupied space strictly closer to 0 with equal probability for each space. The game ends when no coin can be moved or after n turns, whichever comes first. In either case, the player wins the unmovable coins (which all have equal value). For example, if the game ends with coins in positions $\{0, 1, 2, 4, 6\}$ then the player wins 3 coins since the coins in positions 0, 1, and 2 are unmovable.

Let $E(S, t)$ be the expected winnings when following the optimal strategy for a position with coins in locations given by the set S and t turns left. Write a recurrence (including the base cases) for $E(S, t)$. You may use the following functions.

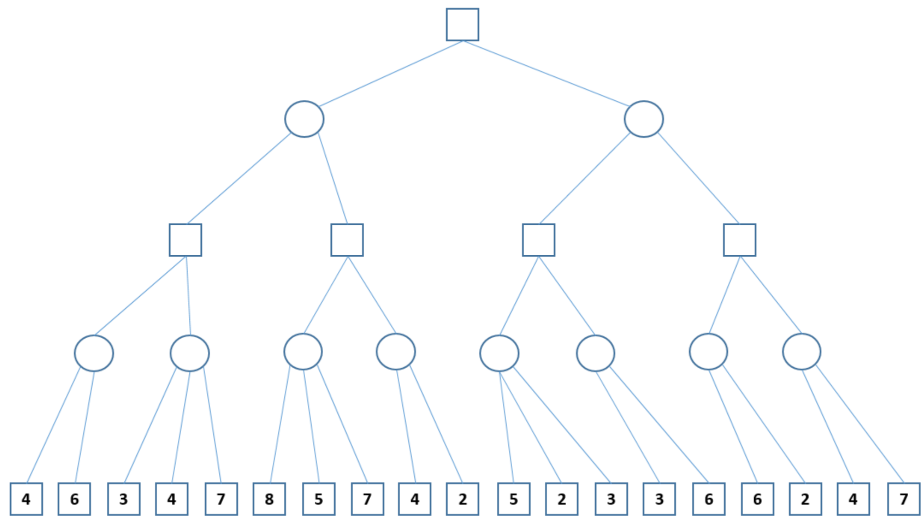
- unmovable(S): when S is the set of positions of coins, returns the subset of those positions containing unmovable coins (so, for example, unmovable($\{0, 1, 4\}$) $= \{0, 1\}$)
- moveable(S): when S is the set of positions of coins, returns the subset of those positions containing movable coins (so, for example, movable($\{0, 1, 4\}$) $= \{4\}$)
- moveto(S, p): when S is the set of positions of coins and p is the position of a movable coin, returns the set of positions that coin could move to (so, for example, moveto($\{0, 2, 4, 6\}, 4$) $= \{1, 3\}$).

Problem 5 (10 points): Let G be a finite, impartial, normal, combinatorial game and let G' be an option of G. Can G and G' both be N-positions? Can they both be P-positions? Explain your answers.

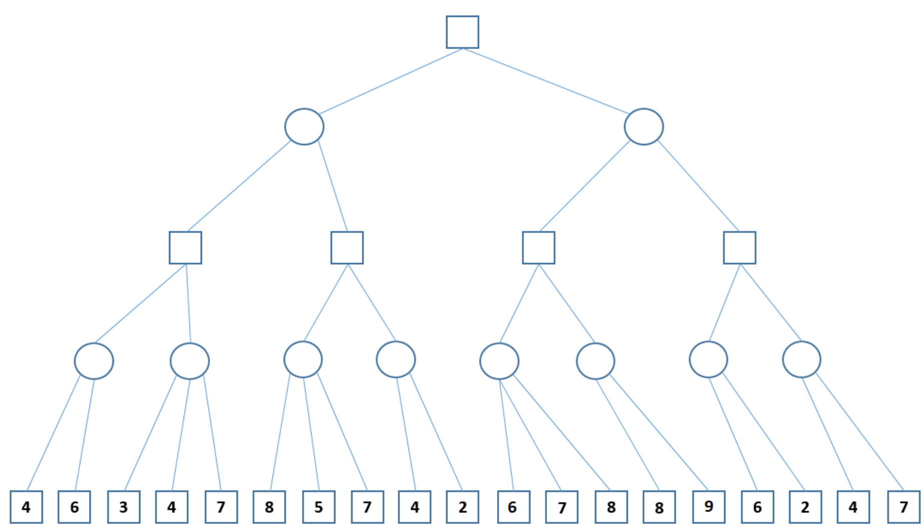
Problem 6 (10 points): Compute the Minimax values for the given game tree (squares are max nodes and circles are min nodes).



Problem 7 (10 points): Illustrate the operation of Alpha-Beta pruning on the given game tree by showing the (α, β) window passed to each non-leaf and which branches are pruned (you need not indicate the values returned).



Problem 8 (10 points): Illustrate the operation of Scout on the given game tree by showing the null window that is passed to the right child of the root and then the results of the search with that null window: show the (α, β) windows passed to nodes in the right subtree of the root and which branches are pruned in that subtree. Indicate whether the right child of the root will be re-searched as a result of the initial null-window search. Note that while the left subtree is the same as the previous two, the right subtree is different.



Problem 9: (10 points) Find a saddle point in mixed strategies for the constant-sum game with payoff matrix

$$\begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix}$$

Problem 10: (10 points) Consider the constant sum game with the following payoff matrix.

$$\begin{pmatrix} 3 & 1 & 3 \\ 4 & 1 & 0 \\ 1 & 2 & 4 \end{pmatrix}$$

For each of the following pairs of mixed strategies, determine if it is a saddle point, and if not, find player I's best response to the given strategy for player II (find the best response to Y_i).

(a) $X_1 = (0\ \frac{1}{4}\ \frac{3}{4}), Y_1 = (\frac{1}{4}\ \frac{3}{4}\ 0)$

(b) $X_2 = (0\ \frac{1}{2}\ \frac{1}{2}), Y_2 = (\frac{1}{4}\ \frac{1}{2}\ \frac{1}{4})$