

Combinatorial Games	Nim, Kayles	Chess, Checkers, Go	Backgammon, Yahtzee	Poker	Roshambo	Starcraft
Combinatorial Game:						
two-player	✓	✓	✓	✗	✓	✗
turn-based	✓	✓	✓	✓	✗	✗?
non-stochastic	✓	✓	✗	✗		✗
perfect information ↳ know all actions and outcomes for each player	✓	✓	✓	✗		✗
impartial	✓	✗	✗			
normal last move wins	(✓)					
misere last move loses						
finite bounded # turns	✓	✓	infinite ✓			

Finite Combinatorial Games

Divisors : Start with 1...n, players take turns taking a number with remaining divisors; opponent gets all the remaining divisors. Game is over when no moves remain; winner is player with higher sum (draw if =)

parent move child

root = initial pos

Game Tree

P1 P2 1 2 3 4 5 6

P1 P2

-1 remaining

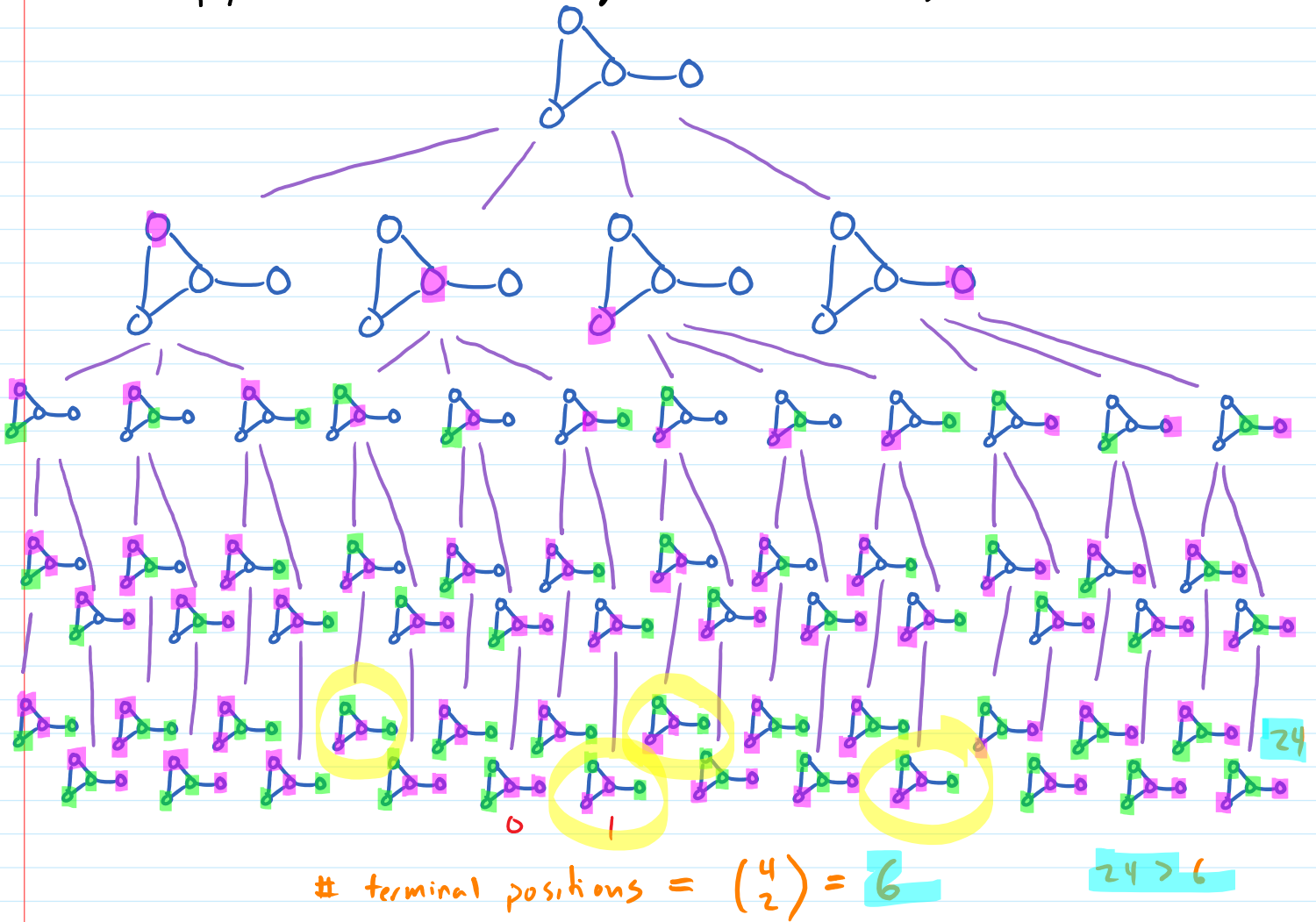
3 1 1 2 4 5 6 4 1 2 3 5 6 5 1 1 2 3 4 6 6 1 2 3 4 5

P1 2 3 1 6 4 5 3 2 1 4 5 6 3 2 1 6 4 5 4 3 1 2 6 5 5 2 4 1 3 6 5 2 3 1 6 4

leaves = game over (terminal)

+1 P1 wins
0 draw
-1 P2 wins

Graph: take turns coloring a vertex in a graph with your color player who covers the most edges wins (draw if =)

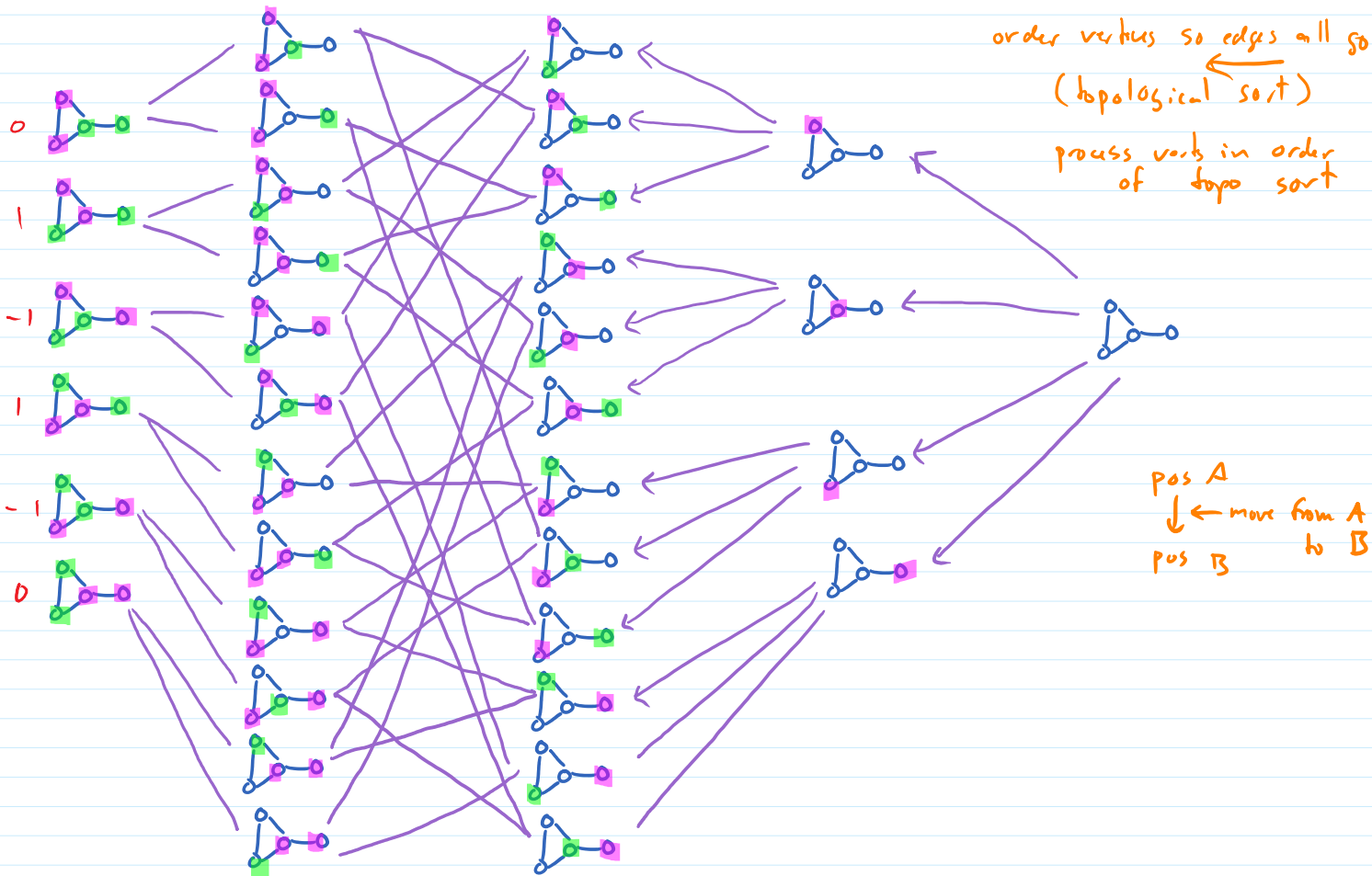


Dynamic Programming

Order positions by maximum distance to end.

Determine winner of distance 0 positions (end) by

Use recursive formula to determine value of other positions in order of



https://en.wikipedia.org/wiki/Game_complexity

Kalah : pos = arrangement of seeds + turn

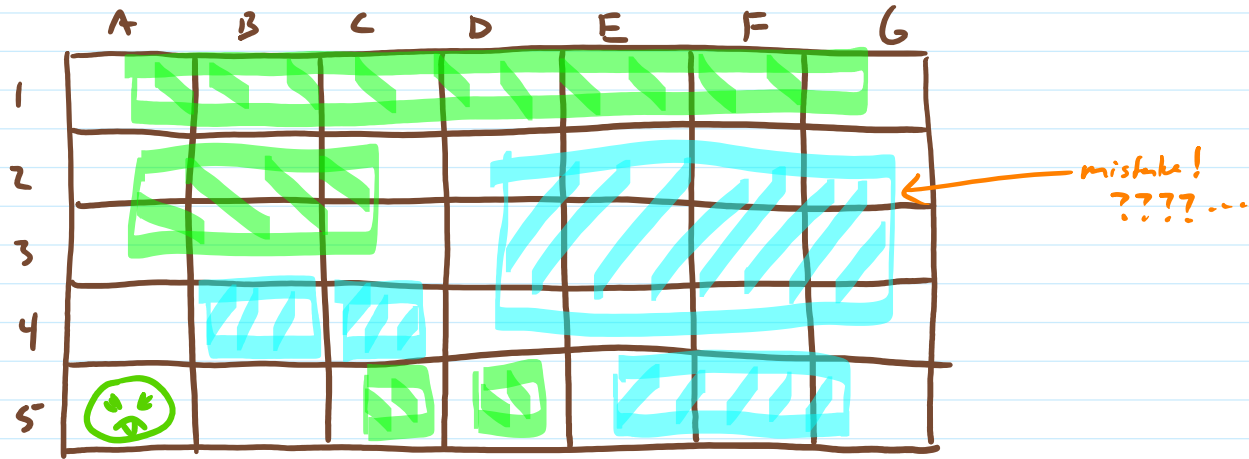
↑
48 seeds
in
14 pits

$$\binom{61}{44} = \binom{48+14-1}{44}$$

Chomp

Play on $m \times n$ grid. Take turns selecting a remaining cell, remove all above and to right.

Last move loses combinatorial, impartial, misere



Outcome class = who has winning strategy

N	next player wins
P	prev player wins

Any position in a finite, impartial, normal or misere game is N or P

in order of max moves to end

