

given any pos, P want
 $E[p] =$ expected payout given that game has reached p

<https://play.golang.org/p/3IFJkluUVc>
<https://play.golang.org/p/ls4evuDNNt>

Analysis of 1-player Finite Probabilistic Games

$E(\text{pos})$ = expected winnings having reached position pos

For final positions pos, $E(\text{pos})$ determined by rules

For non-final choice positions

$$E(\text{pos}) = \max_{\text{choice } c} E[\text{next}(\text{pos}, c)]$$

→ position reached by making choice c from position pos

For non-final random event positions

$$E(\text{pos}) = \sum_{\text{outcome } \sigma} P(\sigma) \cdot E[\text{next}(\text{pos}, \sigma)]$$

for every terminal position pos
 $E(\text{pos}) \leftarrow \text{payoff}(\text{pos})$

for every nonterminal position pos in reverse order of topological sort

if pos is a choice position
 $\text{max} \leftarrow -\infty$
 $\text{argmax} \leftarrow \text{NIL}$
 for every choice c
 $e \leftarrow E[\text{next}(\text{pos}, c)]$
 if $e > \text{max}$
 $\text{max} \leftarrow e$
 $\text{argmax} \leftarrow c$

$E(\text{start}) = \text{value of game}$

$E(\text{pos}) \leftarrow \text{max}$
 $\text{opt}(\text{pos}) \leftarrow \text{argmax}$

else

$e \leftarrow 0.0$
 for every outcome σ
 $e \leftarrow e + P(\sigma) \cdot E[\text{next}(\text{pos}, \sigma)]$
 $E(\text{pos}) \leftarrow e$

Coins: Start with n coins.

On each turn, flip as many of your remaining coins as you wish.

If $\#T \geq \#H$, lose all the T

Else earn $\#H$ points

Win at X points

If $\#1 = \#2$, lose all the 1
Else earn $\#H$ points
Win at X points
Lose if no coins left and $< X$ points

anchor: position at start of turn (before 1st roll)

component: positions reachable from one anchor w/o going through another

number of anchors: $7^6 \cdot 28^2 \cdot 27 \cdot 3^3 \cdot 15 \approx 1 \text{ trillion anchors}$

7^6 : 7 ways for each of 1s, ..., 6s
 28^2 : unused or 0, 5...30 for 3k, 4k
 27 : same as 6 but can't be 0
 3^3 : unused nonzero FN, SS, LS
 15 : unused 0, 50, ..., 1250

1600 pos/anchor
 ||
 ~ 1.6 quadrillion positions

modification: $E(\text{pos}) =$ expected future score from pos

For nonterminal choice positions

$$E(\text{pos}) = \max_{\text{choice } c} E[\text{next}(\text{pos}, c)] + \text{score}(\text{pos}, c)$$

For nonterminal random event positions

$$E(\text{pos}) = \sum_{\text{outcome } \sigma} P(\sigma) \cdot (E[\text{next}(\text{pos}, \sigma)] + \text{score}(\text{pos}, \sigma))$$

anchors = $2^{12} \cdot 3 \cdot 6^4 \approx 750 \text{ million anchors}$

2^{12} : used/unused
 3 : unused 0 nonzero
 6^4 : upper total

$\cdot 1600$
 $\approx 2 \text{ billion}$

"position" = minimal info that affects strategy

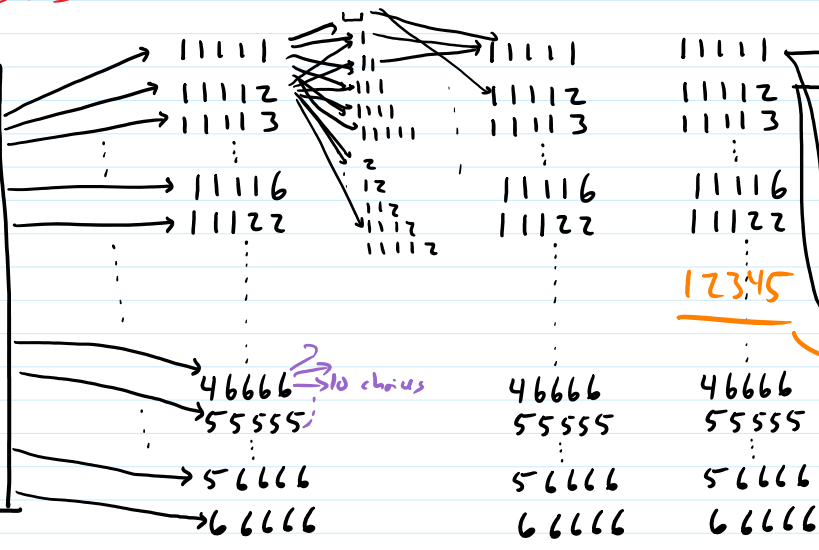
so < 1 hour if 1million pos/sec

Yahtzee Graph

66633

Aces	10
Deuces	2
Tris	9
Fours	12
Fives	15
Sixes	18
3 Kind	25
4 Kind	0
Full House	25
S Straight	30
L Straight	-
Chance	15
Yahtzee	-

anchor



252 positions 462 pos

252 462 252

12345

Aces	1
Deuces	2
Tris	9
Fours	12
Fives	15
Sixes	18
3 Kind	25
4 Kind	0
Full House	25
S Straight	30
L Straight	0
Chance	15
Yahtzee	-

Aces	1
Deuces	2
Tris	9
Fours	12
Fives	15
Sixes	18
3 Kind	25
4 Kind	0
Full House	25
S Straight	30
L Straight	15
Chance	-
Yahtzee	-

Aces	1
Deuces	2
Tris	9
Fours	12
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Sixes	18
3 Kind	25
4 Kind	0
Full House	25
S Straight	30
L Straight	-
Chance	15
Yahtzee	0

15

Aces	1
Deuces	2
Tris	9
Fours	12
Fives	15
Sixes	18
3 Kind	25
4 Kind	0
Full House	25
S Straight	30
L Straight	-
Chance	15
Yahtzee	50

15