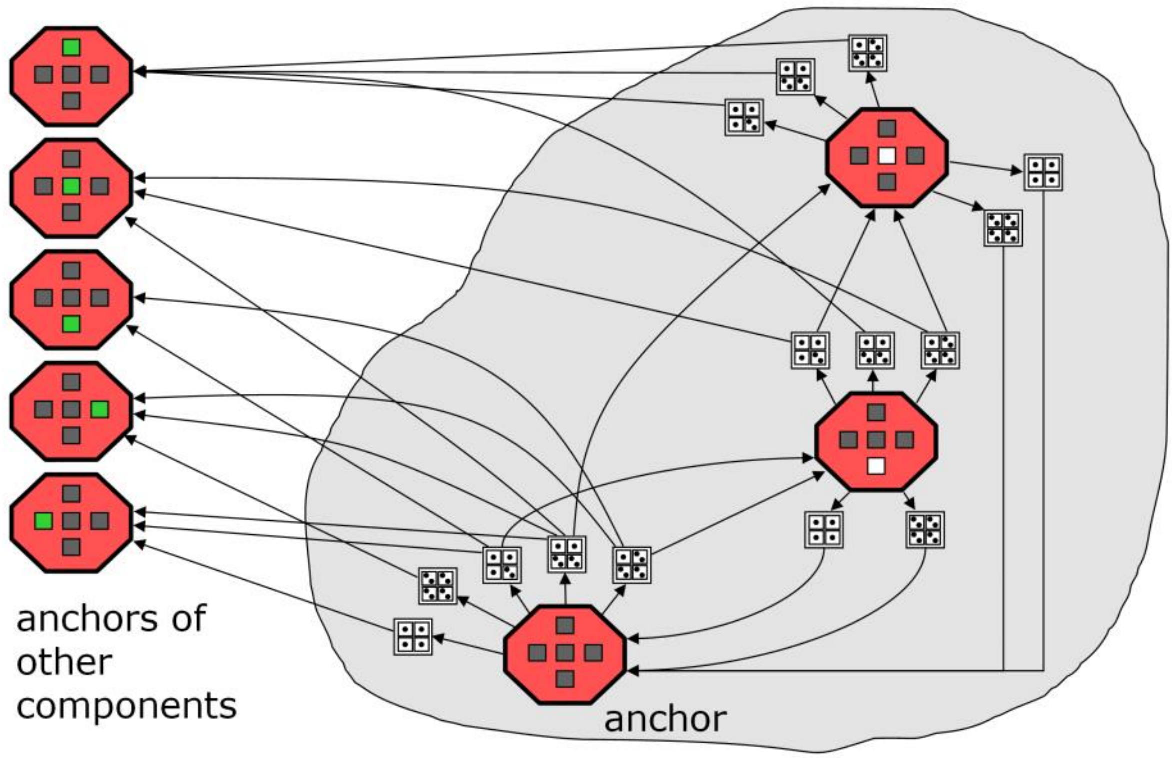
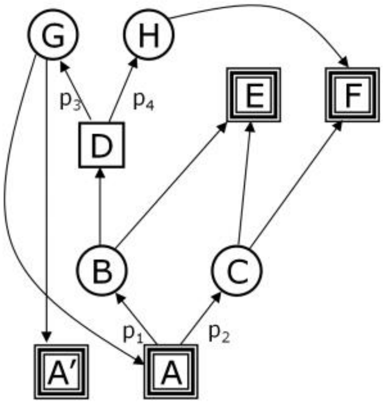
Can't Stop

Can't Stop Graph



Numerical Analysis



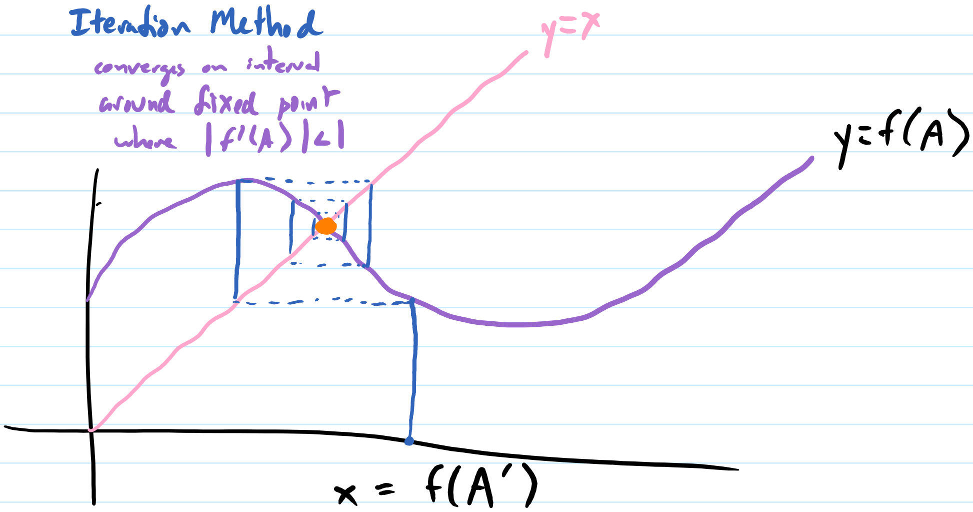
- Make a copy of anchor
- component is now a DAG
- Guess value of f(A′)
- f(A) is a function of f(A′)
- Want fixed point

- Function is piecewise linear and continuous
- Fast convergence from Newton's method

https://en.wikipedia.org/wiki/Newton%27s_method

Iteration Method

converges on interval around fixed point where $|f'(A)| < 1$



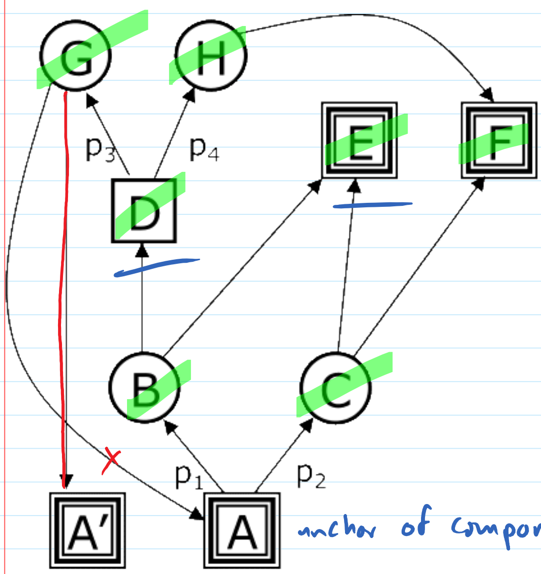
$y = f(x)$

solve for x s.t. $f(x) = x$

find x s.t. $f(x) - x = 0$

Newton's Method converges if f'' is continuous, $f'(x) \neq 0$ on some open neighborhood of x^* and initial guess is "close enough"

x_1
x_2
x_0
$y = f(x) - x$



anchors of later components

anchor of component to solve

$v(C) = \min(v(E), v(F))$

$v(H) = \min(v(F)) = v(F)$

$v(G) = v(A')$

$v(D) = p_3 \cdot v(G) + p_4 \cdot v(H)$

$v(B) = \min(v(D), v(E))$

$v(A) = p_1 \cdot v(B) + p_2 \cdot v(C)$

$= p_1 \cdot \min(v(D), v(E)) + p_2 \cdot \min(v(E), v(F))$

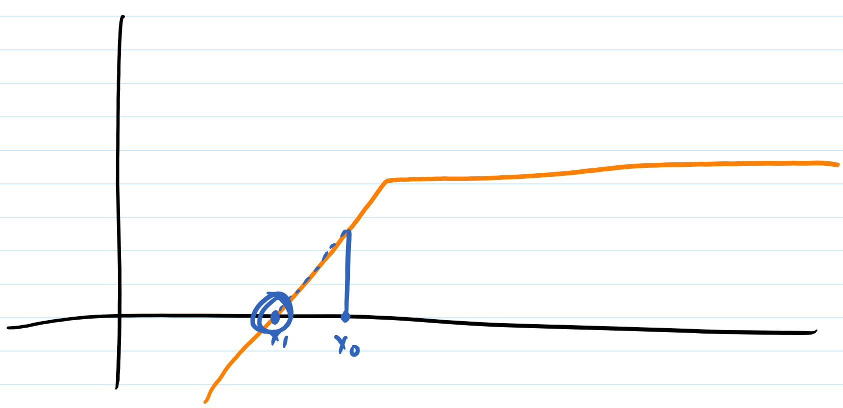
$\vdots$ some linear fns of $v(E), v(F)$

$= p_1 \cdot \min(p_3 \cdot v(A') + c_3, c_2) + c_1$

$$= \begin{cases} p_1 \cdot p_3 \cdot v(A') + c_1 & \text{if } p_3 v(A') \le c_2 \\ p_1 \cdot c_2 + c_1 & \text{otherwise} \end{cases}$$

piecewise linear!

makes $f'(x)$ to compute and convergence fast



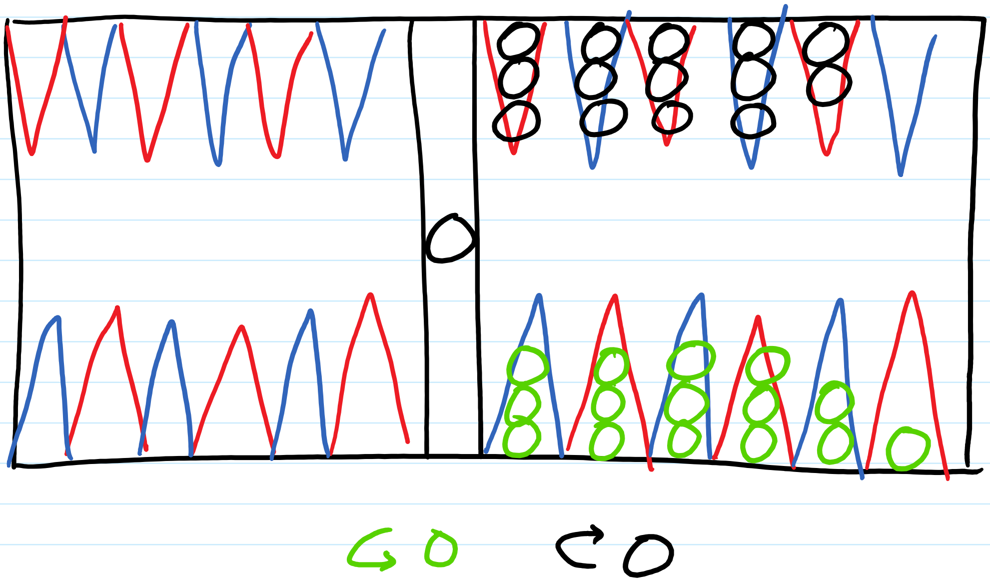
Results for Can't Stop

Results

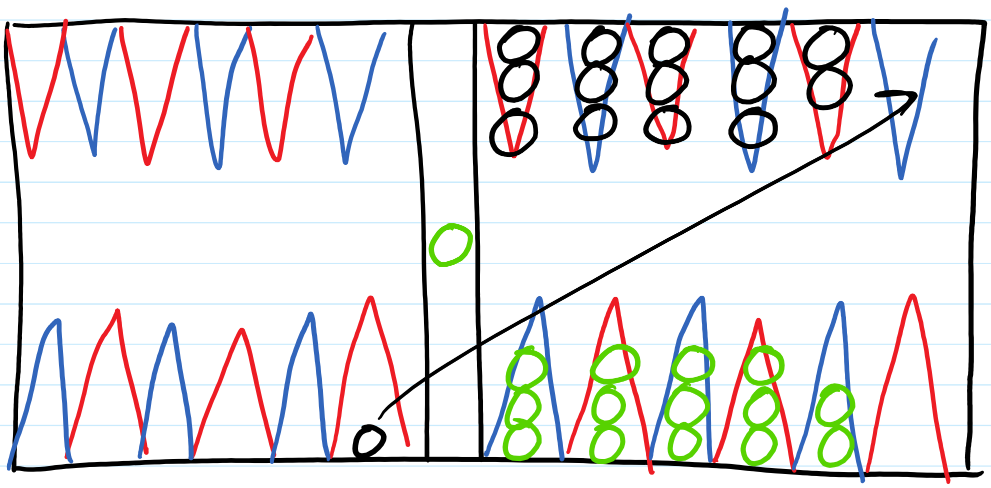
Dice	Size	Graph Size	Time to Solve	Optimal Turns
2	1	225	0.16 sec	1.298
2	2	1,936	0.26 sec	1.347
2	3	9,025	0.55 sec	1.400
3	1	64,372	1.3 sec	1.480
3	2	787,600	3.3 sec	1.722
3	3	4,934,006	10 sec	1.890
4	1	20,802,843	99 sec	2.187
4	2	289,091,584	21 min	2.454
4	3	2,104,663,011	2.6 hr	2.700
5	1	7,105,015,062	19 hr	2.791

- Official game: estimated 1000 years

Backgammon https://cardgames.io/backgammon/

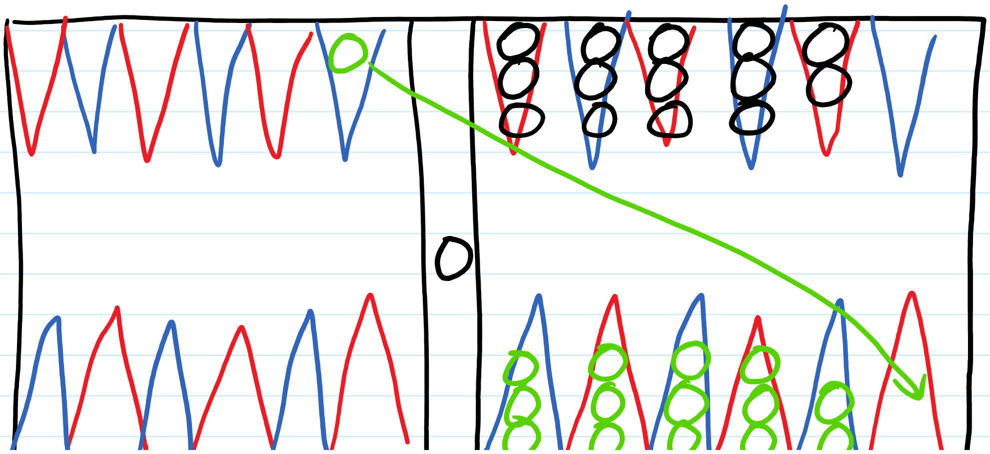


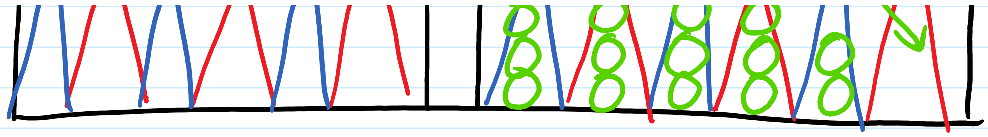
Black rolls 1-6



Green rolls 5-6 4-4 3-6 1-6

Black rolls 1-2 6-3 3-2





Black rolls 6-5 4-3 5-5 2-4 1-6
Green rolls 2-3 1-3 1-2 3-2

so looooong cycles → converges slowly

(and has large state space)

3+ Player Games

$E_n[pos]$ = expected value for player n given game has reached position pos, and assuming other players play optimally

$$= \sum_{\text{outcome } \sigma} P(\sigma) \cdot E_n[next(pos, \sigma)]$$ for all players n   for random events

$$= \max_{\text{choice } c} E_n[next(pos, c)]$$ when player n has a choice

$$= E_n\left[next\left(pos, \operatorname*{argmax}_{\text{choice } c} E_m[next(pos, c)]\right)\right]$$ when player m has a choice, $m \neq n$

Collusion : $E_n[pos]$ incorporates how player n feels about other players winning/losing