S18 Exam #1 Solutions

CPSC 474/574 - Spring 2018 - Exam #1

Write your name and NetID on this cover page and only on this cover page. Please indicate clearly your final answers to each question.

0	1	2	3	4	5	6	7	8	9
N	P	N	N	P	N	N	P	N	N

*0

*1   OO   *1

*2   O.O   OO   *1   *0

*2   O.O   OO   *1   *0

*1   O..O   OO.   OO   *2   *2   *0

*0   OO.   O.O   O.O.   O..O   *2   *1   *1   *2

*3   O..O   O.O   OO   *2   *1   *0

*4   O.O.   OO.   OO   O...O   *1   *2   *0   *3

*5   OO..   OOO.   OO.O   O..O.   O...O

*0   *2   *1   *4   *3

$*3 + *4 + *1$

$$\begin{array}{r} 011 \\ 100 \\ 001 \\ \hline 110 \end{array}$$

$100 \rightarrow$ change to $010 = *3 + *1$

x x x x x x   x x x ~~x~~ x   x

$$E(\{0, \dots, k-1\}, t) = k$$

$$E(S, 0) = |\text{unmovable}(S)|$$

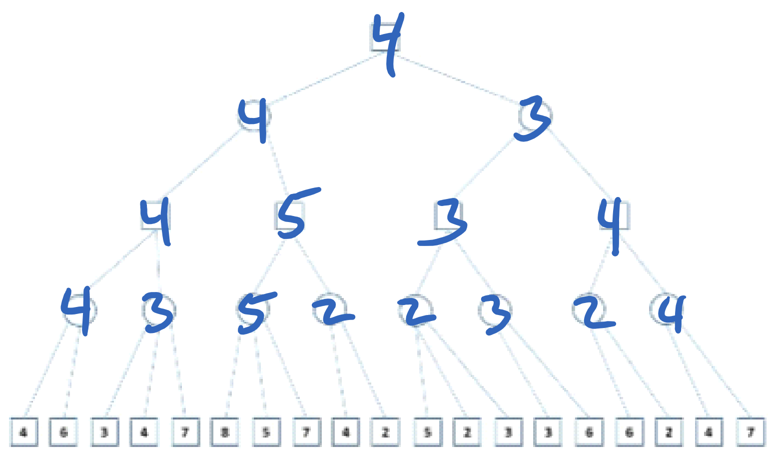
$$E(S, t) = \max_{p \in \text{moveable}(S)} \frac{1}{|\text{move to}(S, p)|} \cdot \sum_{q \in \text{moveto}(S, p)} E(S - \{p\} \cup \{q\}, t-1)$$

Problem 5 (10 points): Let G be a finite, impartial, normal, combinatorial game and let G' be an option of G. Can G and G' both be N-positions? Can they both be P-positions? Explain your answers.

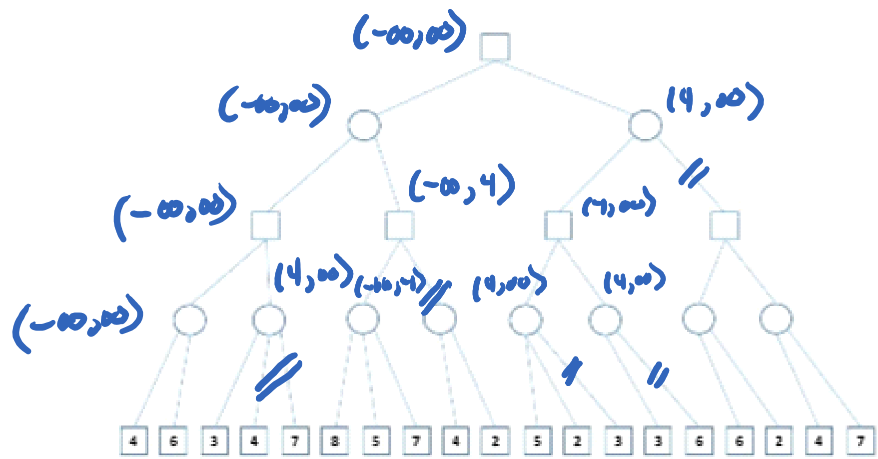
Both N: yes, ex $*2 \to *1$

Both P: no, by def. all moves from P are to N

Problem 6 (10 points): Compute the Minimax values for the given game tree (squares are max nodes and circles are min nodes).

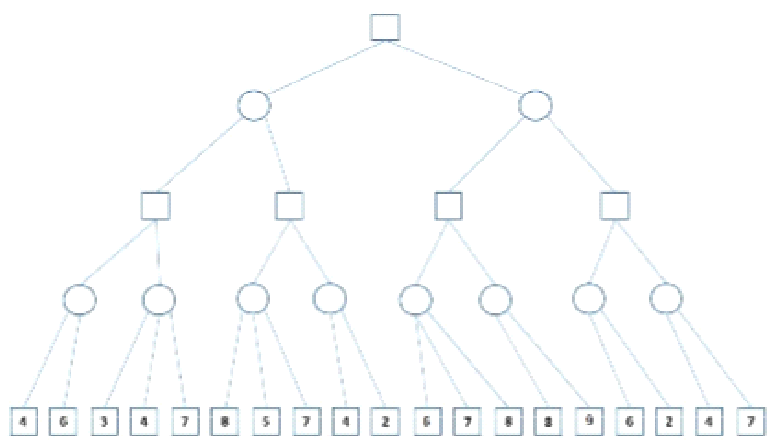


Problem 7 (10 points): Illustrate the operation of Alpha-Beta pruning on the given game tree by showing the (α, β) window passed to each non-leaf and which branches are pruned (you need not indicate the values returned).



Problem 8 (10 points): Illustrate the operation of Scout on the given game tree by showing the null window that is passed to the right child of the root and then the results of the search with that null window: show the (α, β) windows passed to nodes in the right subtree of the root and which branches are pruned in that subtree. Indicate whether the right child of the root will be re-searched as a result of the initial null-window search. Note that while the left subtree is the same as the previous two, the right subtree is different.

OUT OF SCOPE FOR F18



Problem 9: (10 points) Find a saddle point in mixed strategies for the constant-sum game with payoff matrix

$$\begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix}$$

$E(X,1) = x_1 - (1-x_1) = -1 + 2x_1$

$E(X,2) = 4 - 4x_1$

$-1 + 2x_1 = 4 - 4x_1$

$6x_1 = 5$

$x_1 = \frac{5}{6}$

$X^* = (\frac{5}{6} \ \frac{1}{6})$

$Y^* = (\frac{2}{3} \ \frac{1}{3})$

$E(1,Y) = y_1$

$E(2,Y) = 4 - 4y_1 - y_1$

$y_1 = 4 - 5y_1$

$6y_1 = 4$

$y_1 = \frac{2}{3}$

Problem 10: (10 points) Consider the constant sum game with the following payoff matrix.

$$\begin{array}{cc} \begin{pmatrix} 3 & 1 & 3 \\ 4 & 1 & 0 \\ 1 & 2 & 4 \end{pmatrix} & \begin{array}{c} \frac{8}{4} \\ \frac{6}{4} \\ \frac{9}{4} \end{array} \end{array}$$

For each of the following pairs of mixed strategies, determine if it is a saddle point, and if not, find player I's best response to the given strategy for player II (find the best response to Y_i).

(a) $X_1 = (0 \ \frac{1}{4} \ \frac{3}{4}), Y_1 = (\frac{1}{4} \ \frac{3}{4} \ 0)$ saddle point

(b) $X_2 = (0 \frac{1}{2} \ \frac{1}{2} \ ), Y_2 = (\frac{1}{4} \ \frac{1}{2} \ \frac{1}{4})$

$E(X_1,Y_1) = \frac{4}{16} + \frac{3}{16} + \frac{3}{16} + \frac{18}{16} = \frac{28}{16} = \frac{7}{4}$

$E(X_1,1) = \frac{4}{4} + \frac{3}{4} = \frac{7}{4} \geq \frac{7}{4}$ ✓

$E(X_1,2) = \frac{1}{4} + \frac{6}{4} = \frac{7}{4} \geq \frac{7}{4}$ ✓

$E(X_1,3) = \frac{0}{4} + \frac{12}{4} = \frac{12}{4} \geq \frac{7}{4}$ ✓

$E(1,Y_1) = \frac{3}{4} + \frac{1}{4} = \frac{5}{4} \leq \frac{7}{4}$ ✓

$E(2,Y_1) = \frac{4}{4} + \frac{3}{4} = \frac{7}{4} \leq \frac{7}{4}$ ✓

$E(3,Y_1) = \frac{1}{4} + \frac{6}{4} = \frac{7}{4} \leq \frac{7}{4}$ ✓

all inequalities were true, so (X_1, Y_1) is a saddle point

$E(X_2,Y_2) = \frac{6}{8} + \frac{9}{8} = \frac{15}{8} \neq \frac{7}{4}$

not a saddle point

$E(1,Y_2) = \frac{8}{4}$

$E(2,Y_2) = \frac{6}{4}$

$E(3,Y_2) = \boxed{\frac{9}{4}}$

best response $(0 \ 0 \ 1)$