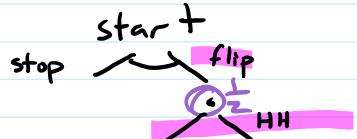


Coin Flipping Game



↗ player choice of subsequent position

flip 2 coins
if HH choose flip again or take winnings
otherwise lose

$V(s)$ = expected payout given game has reached state s (and you play to maximize payout)

$$\max(64, 62.5) = 64$$

$$\begin{aligned} &\text{not } \text{HH} \\ &\frac{1}{4} \cdot 250 + \frac{3}{4} \cdot 0 \\ &= 62.5 \end{aligned}$$

<https://play.golang.org/p/3IFJkluUVc>
<https://play.golang.org/p/lS4evuDNNt>

Finite 1-Player Probabilistic Games

$V(s)$ = expected winnings having reached state s

game over

For terminal states s , $V(s)$ determined by rules

For nonterminal action states

$$V(s) = \max_{\text{action } a} V(\underbrace{\text{next}(s, a)}_{\substack{\text{the state that results from} \\ \text{action } a \text{ in state } s}})$$

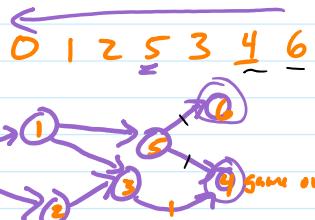
For nonterminal random event states

$$V(s) = \sum_{\text{outcome } \sigma} P(\sigma) \cdot V(\underbrace{\text{next}(s, \sigma)}_{\substack{\text{state resulting from outcome } \sigma \\ \text{in state } s}})$$

for every terminal state s

$\underbrace{V[s]}_{\substack{\text{for example,} \\ \text{closest to end of game} \\ \text{to furthest}}} \leftarrow \text{payout}(s)$

(for example)
closest to end of game
to furthest



→ for every nonterminal state s in order of topological sort

if s is an action position

$\max \leftarrow -\infty$

$\text{argmax} \leftarrow \text{NIL}$

for every action a

$$v \leftarrow V[\text{next}(s, a)] + R(s, a)$$

reward for choosing
 a in state s

if $v > \max$

$\max \leftarrow v$

$\text{argmax} \leftarrow a$

$$V[s] \leftarrow \max$$

$$\text{OPT}[s] \leftarrow \text{argmax}$$

else

$$v \leftarrow 0.0$$

for every outcome σ

$$v \leftarrow v + P(\sigma) \cdot (V[\text{next}(s, \sigma)] + R(s, \sigma))$$

$$V[s] \leftarrow v$$

finite game = directed acyclic
graph

topo sort = ordering of visits
so edges all go →

anchor: states at start of turn

component: states reachable from one position w/o going through another

number of anchors: score in each category

$$2^{12} \cdot 64 \cdot 3 = \frac{3}{4} \text{ million TMI!}$$

used/unused ↑ up to total unused, 0, or 50
1-6, 3k, ..., C ±1 minute

modification: $V(s) =$

For nonterminal action positions

$$V(s) = \max_{\text{action } a} V(\underline{\text{next}(s, a)}) + \text{score}(s, a) \approx 50 \text{ years}$$

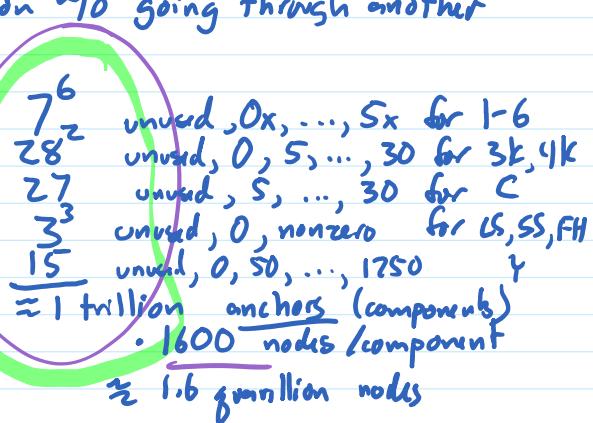
$\hookrightarrow 0$ if s in middle of turn
determined by final roll + cat chosen
at end

For nonterminal random event positions

$$V(s) = \sum_{\sigma} P(\sigma) \cdot \left(V(\underline{\text{next}(s, \sigma)}) + \text{score}(s, \sigma) \right)$$

$\hookrightarrow 0$

anchors =



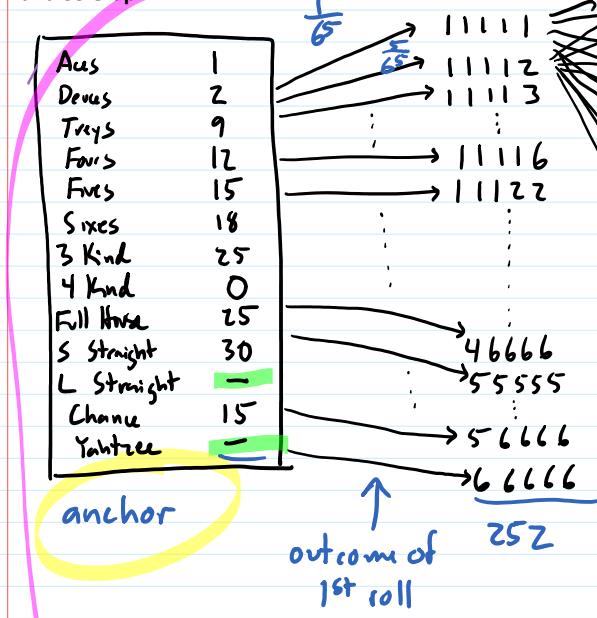
$\approx 1.6 \text{ billion nodes}$

$\approx 1 \text{ billion/sec}$

$1.6 \text{ billion seconds}$

$\approx 50 \text{ years}$

Yahtzee Graph



1/6

6/6

outcome of
1st roll

which die
to keep

462

252

252

end of
turn

component

Aces	1
Duos	2
Treys	9
Fours	12
Fives	15
Sixes	18
3 Kind	25
4 Kind	0
Full House	25
S Straight	30
L Straight	15
Chance	-
Yahtzee	-

anchors
of other
components

Aces	1
Duos	2
Treys	9
Fours	12
Fives	15
Sixes	18
3 Kind	25
4 Kind	0
Full House	25
S Straight	30
L Straight	15
Chance	-
Yahtzee	-

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Yahtzee	-

Hoot Owl Hoot

what captures state
how many states are there
how to simplify
what order to solve in

39 pos + nest
6 owls
14 sun pos

14 sun cards + 6 each of each color
6 colors

4K LS + 2 rerolls
1 1 1 2 2 ? FH or 111

LS 3 6 6 6 6 4K or 6 ?
end of turn