Linear Programming

	F	C	S
F	0.30	0.25	0.20
C	0.26	0.33	0.28
S	0.28	0.30	0.33

find $\underset{\sim}{X} = (x_1\ x_2\ x_3)$

Maximize v
subject to

$E(X,1) = 0.30x_1 + 0.26x_2 + 0.28x_3 \geq v$

$E(X,2) = 0.25x_1 + 0.33x_2 + 0.30x_3 \geq v$

$E(X,3) = 0.20x_1 + 0.28x_2 + 0.33x_3 \geq v$

$x_1 + x_2 + x_3 = 1$

$x_1, x_2, x_3 \geq 0$

add c to all entries to ensure that $v > 0$

$v \geq 0.2$

$0 \leq x_i \leq 1$

divide by v let $p_i = \frac{x_i}{v}$
multiply by -1

$p_i \leq \frac{1}{v}$

$p_i \leq 5$

$a1 = (-A)^T$

$-0.3p_1 - 0.26p_2 - 0.28p_3 \leq -1$

$-.25p_1 - 0.33p_2 - 0.3p_3 \leq -1$

$-.2p_1 - 0.28p_2 - 0.33p_3 \leq -1$

b-ub

$0 \leq p_i \leq 5$ bounds

min $p_1 + p_2 + p_3$ subject to

c

$= \frac{x_1}{v} + \frac{x_2}{v} + \frac{x_3}{v} = \frac{x_1+x_2+x_3}{v} = \frac{1}{v}$

lin prog package returns p_i and $\frac{1}{v}$
we convert back to x_i, v

II

minimize v subject to

$E(1,Y) = 0.3y_1 + 0.25y_2 + 0.2y_3 \leq v$

$E(2,Y) =$

$E(3,Y) =$

divide by v

$q_i = \frac{y_i}{v}$

minimize $-q_1 + -q_2 + -q_3$

$= -\frac{1}{v}$

for Java

$-0.3p_1 - 0.26p_2 - 0.28p_3 + s_1 = -1$

for $0 \leq s_1 \leq 1$

$-0.25p_1 - 0.33p_2 - 0.3p_3 + s_2 = -1$

$0 \leq s_2 \leq 1$

⋮

Blotto

	MI	AZ	CO	NH
battleground value	4	3	2	1
P1	4	2	2	2
P2	1	2	4	3

$4 + \frac{3}{2} = 5\frac{1}{2}$ WINNER

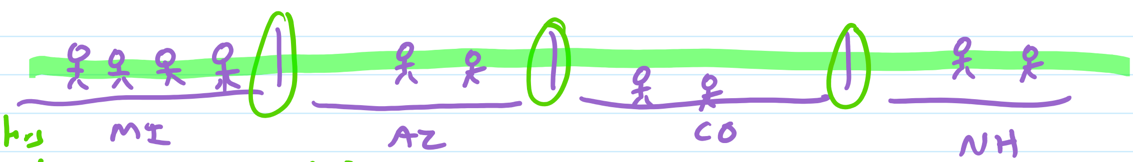
$\frac{3}{2} + 2 + 1 = 4\frac{1}{2}$

strategy: allocation of 10 units to 4 locations

1 2 4 3

$$\begin{array}{c} \\ 10\ 0\ 0\ 0 \\ 9\ 1\ 0\ 0 \\ 4\ 2\ 2\ 2 \\ \vdots \\ 0\ 0\ 0\ 10 \end{array} \begin{array}{ccccc} 10\ 0\ 0\ 0 & 9\ 1\ 0\ 0 & 9\ 0\ 1\ 0 & \cdots\cdots & 0\ 0\ 0\ 10 \\ \end{array}$$

$$\begin{pmatrix} \frac{1}{2} & & & & \\ & \frac{1}{2} & 1 & & \\ & & & 1 & \\ & & & & \\ 0 & 0 & 0 & & \frac{1}{2} \end{pmatrix}$$



string of 13 characters with 10 (person) and 3 |

$\#$ of $\binom{13}{3} = 286$

Non-constant sum

X, Y is equilibrium if and only if
$$E_I(i, Y) \leq E_I(X, Y) \quad \text{for all } i$$
$$E_{II}(X, j) \leq E_{II}(X, Y) \quad \text{for all } j$$

	F	O
F	4, 1	0, 0
O	0, 0	1, 3

$X = \left(\frac{3}{4} \quad \frac{1}{4}\right) \qquad Y = \left(\frac{1}{5} \quad \frac{4}{5}\right)$

value of game for I — value of game II

Non linear programming : maximize $E_I(X, Y) + E_{II}(X, Y) - p - q$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} x_i \cdot y_j \cdot a_{ij}$$

non linear

subject to

$$A = \begin{pmatrix} \frac{3}{2} & -1 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & \frac{5}{4} \\ 1 & -1 & 0 \\ 0 & 1 & \frac{5}{4} \end{pmatrix}$$