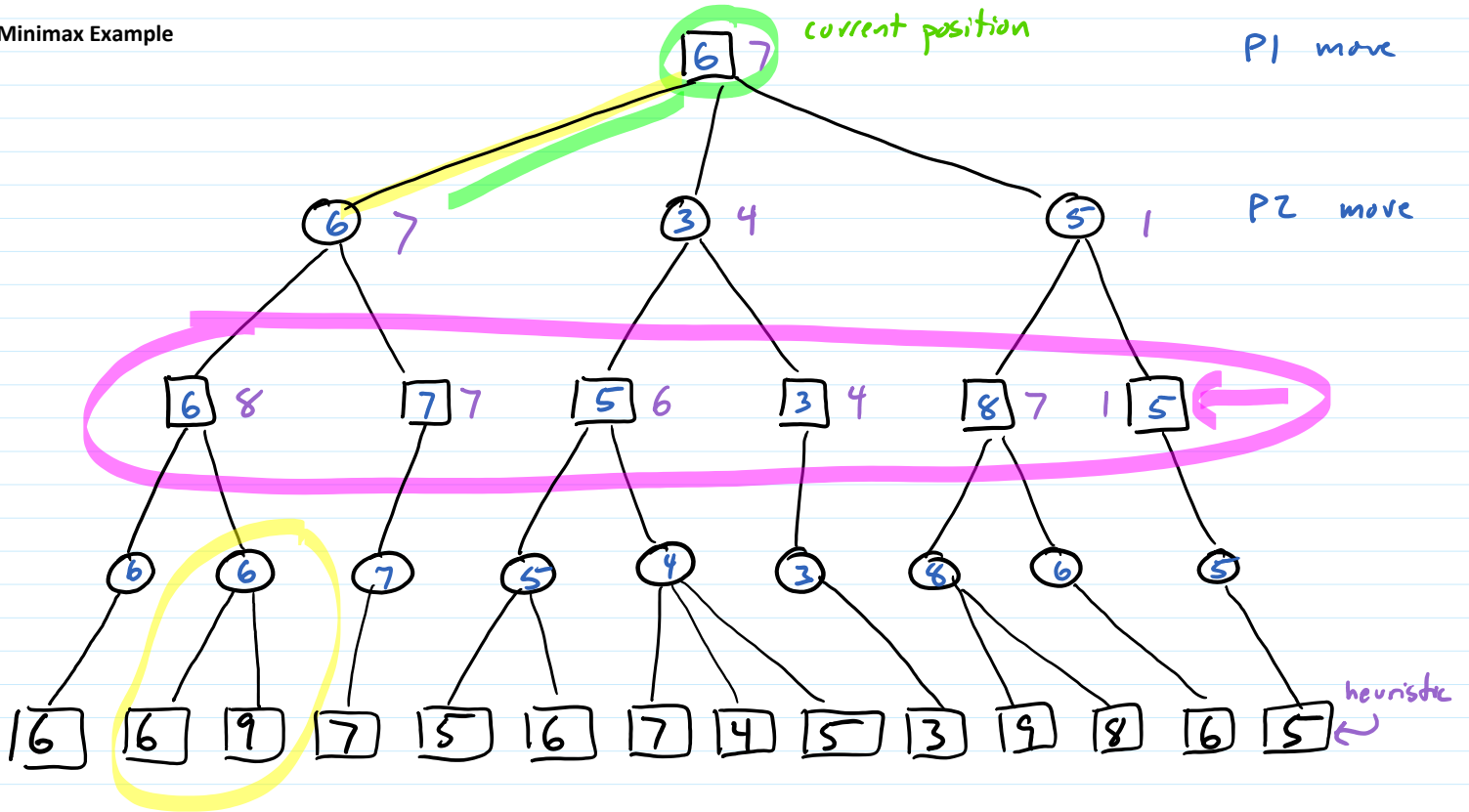


## Calendar

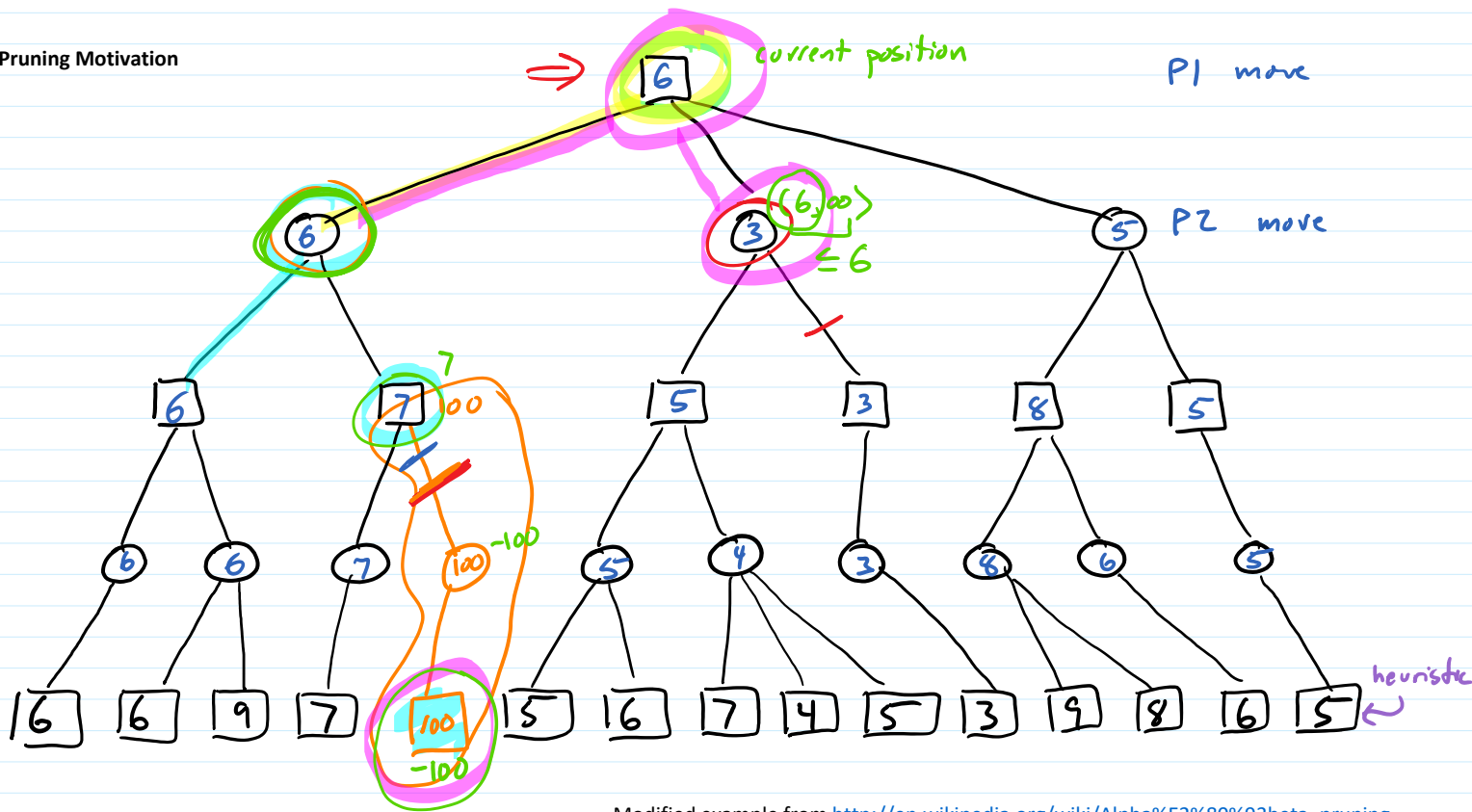
|     |                  |                            |
|-----|------------------|----------------------------|
| Oct | 14               | P3 (Blotto) due            |
|     | 21               | Q2 Simultaneous games      |
|     | 28               | P4 (search) due            |
| Nov | 6 <sup>Fri</sup> | Q3 Search quiz             |
|     | 11               |                            |
|     | 18               | P5 (RL) due                |
|     | 23-27            |                            |
| Nov | 30/Dec           | videos due / responses due |
| Dec | 4                |                            |
|     | 10               | Final projects due         |

Minimax Example



Modified example from [http://en.wikipedia.org/wiki/Alpha%E2%80%93beta\\_pruning](http://en.wikipedia.org/wiki/Alpha%E2%80%93beta_pruning)

Pruning Motivation



Modified example from [http://en.wikipedia.org/wiki/Alpha%E2%80%93beta\\_pruning](http://en.wikipedia.org/wiki/Alpha%E2%80%93beta_pruning)

Minimax w/ Transposition Table

Minimax (pos, h, d) <sup>depth bound</sup>

+1 P1 win  
0 draw  
-1 P2 win

If pos is terminal, return value(pos) <sup>determined by rules</sup>

If depth == 0, return h(pos)

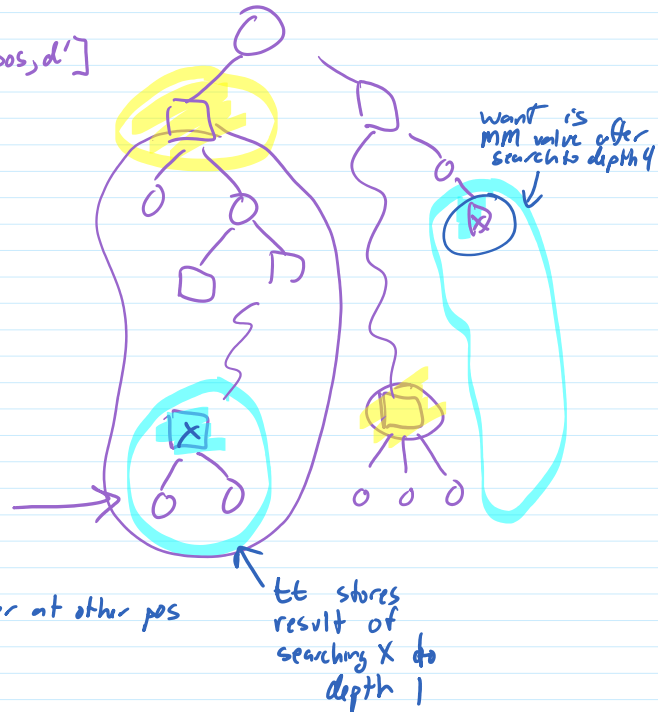
If (pos, d') for some  $d' \geq d$  then return tt[pos, d']

Else if pos is P1's turn then

tt[pos, d] =  
return  $\max_{pos \rightarrow pos'} MM(pos', h, d-1)$

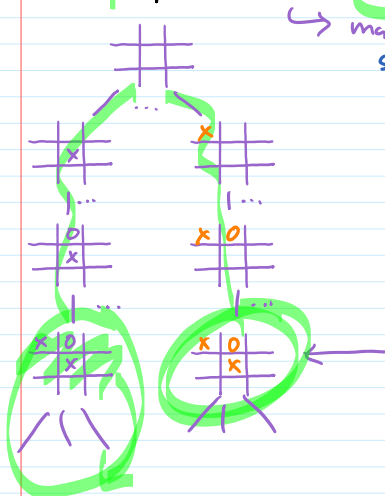
Else

tt[pos, d] =  
return  $\min_{pos \rightarrow pos'} MM(pos', h, d-1)$



Transposition Table - (like memo in memoization)

map from positions  $\rightarrow$  values  
save values for positions in case they reappear at other pos in tree



need replacement policy to avoid t.t. getting too big

stop storing new things?  
replace least recently used item?

(cache replacement policies)

# Alpha-Beta Pruning

range of values we care about

Alpha-Beta ( $p, \alpha, \beta, h, \text{depth}$ ) returns

postconditions

value ( $p$ ) if  $p$  is terminal  
 $h(p)$  if  $\text{depth} = 0$

$MM(p, h, \text{depth})$  if  $\alpha \leq MM(p, d, h) \leq \beta$

lower bound  $\geq \beta$  on  $MM(p, d, h)$  if  $MM(p, d, h) \geq \beta$

upper bound  $\leq \alpha$  on  $MM(p, d, h)$  if  $MM(p, d, h) \leq \alpha$

if  $\text{depth} = 0$  then return heuristic( $p$ )

if  $p$  is terminal then return value( $p$ )

if  $p$  is a max position

$a \leftarrow -\infty$

for each position  $p'$  reachable in one move from  $p$  and while  $\alpha < \beta$

$a \leftarrow \max(a, AB(p', \alpha, \beta, h, \text{depth}-1))$

$\alpha \leftarrow \max(\alpha, a)$

return  $a$

else

$b \leftarrow \infty$

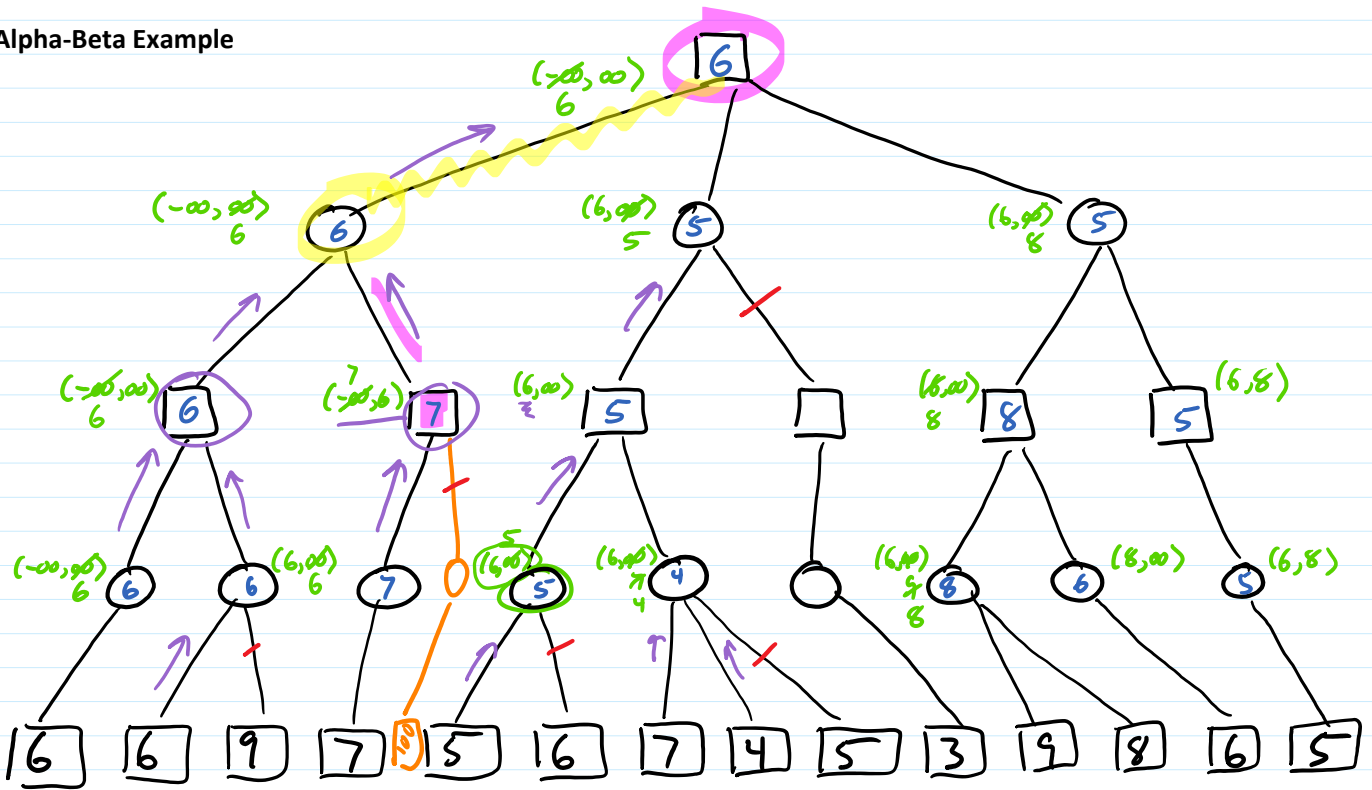
for each position  $p'$  reachable in one move from  $p$  and while  $\alpha < \beta$

$b \leftarrow \min(b, AB(p', \alpha, \beta, h, \text{depth}-1))$

$\beta \leftarrow \min(\beta, b)$

return  $b$

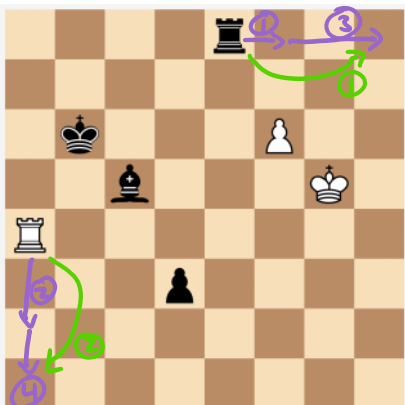
# Alpha-Beta Example



Modified example from [http://en.wikipedia.org/wiki/Alpha%20%80%93beta\\_pruning](http://en.wikipedia.org/wiki/Alpha%20%80%93beta_pruning)

## Transposition Table

Positions may be reachable by multiple sequences of moves



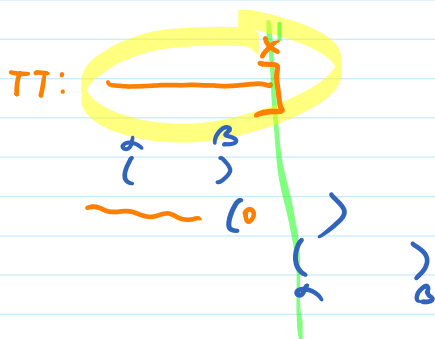
Keep table of values for all positions examined in tree

Keys: positions

Values: (value/bound, move, depth)

Add check at start of A-B

if pos present and searched depth  $\geq$  desired depth  
 if value is exact, return value  
 else if upper bound  $\leq \alpha$  return  $\alpha$   
 else if lower bound  $\geq \beta$  return  $\beta$



Save returned values in table

if value  $\geq \beta$  store lower bound  
 elif value  $\leq \alpha$  store upper bound  
 else store exact value

Scout Example

