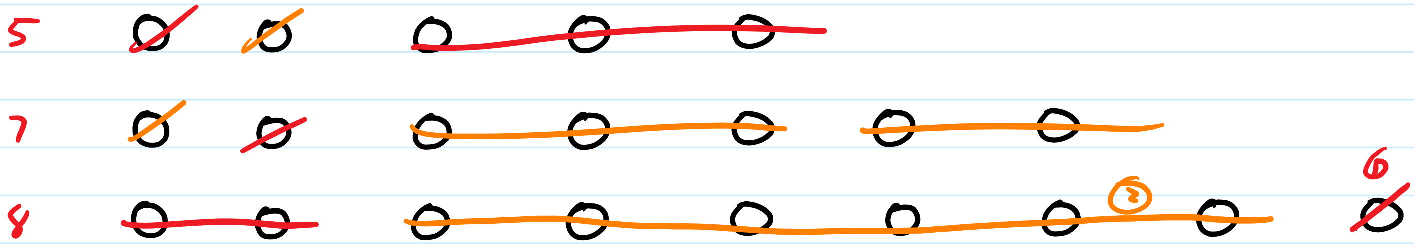
Nim



Start with rows of $n_1, n_2, \ldots, n_k$ stones

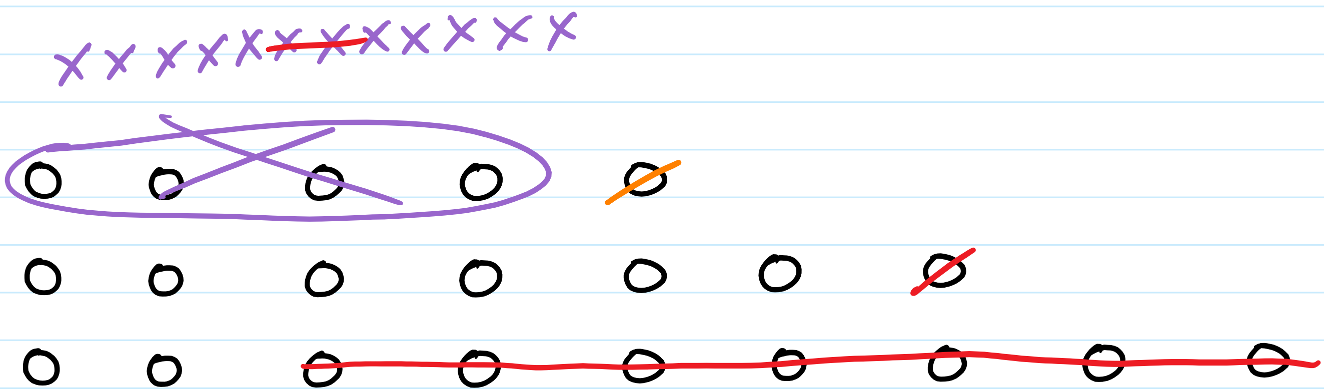
$k = 3$

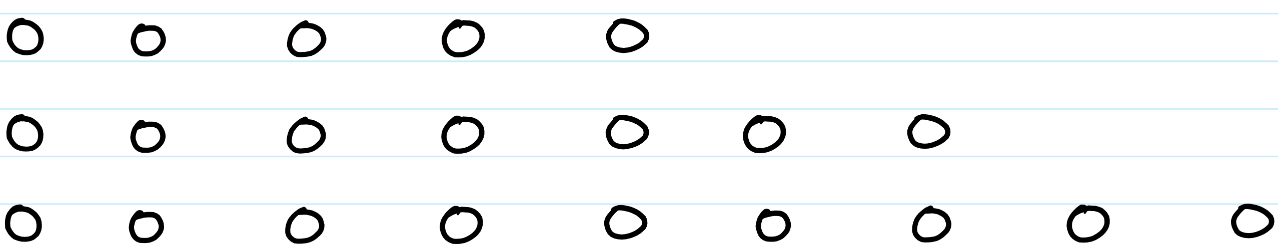
$n_1 = 5$
$n_2 = 7$
$n_3 = 9$

On each turn, take as many stones as you wish from one row

$$\begin{array}{r} 0101 \\ 0111 \\ 1000 \\ \hline 1010 \end{array} \quad 0010$$

If no possible moves, you lose





Kayles

0 5 10 15 20 25 30 35
0, 1, 2, 3, 1, 4, 3, 2, 1, 4, 2, 6, 4, 1, 2, 7, 1, 4, 3, 2, 1, 4, 6, 7, 4, 1, 2, 8, 5, 4, 7, 2, 1, 8, 6, 7

X ~~X X~~ X X ~~X~~ X ~~X~~ X ~~X~~ X X ~~X X~~ X

Start with row of n pins n = 15

On each turn, take 1 or 2 adjacent pins

If no possible moves, you lose

X X X X X X X X X X X X X X X

X X X X X X X X X X X X X X X

Combinatorial Games

	Nim, Kayles	Chess, Checkers, Go	Backgammon, Yahtzee	Poker	Roshambo	Starcraft
Combinatorial Game:						
two-player	✓	✓				✗
turn-based	✓	✓			✗	✗
non-stochastic (no chance)	✓	✓	✗	✗		✗
perfect information	✓	✓		✗		✗
normal last move wins	✓					
misere last move loses						
finite	✓					
impartial all moves available to either player	✓	✗				

https://xkcd.com/1002/

Sprague-Grundy Theorem: every finite, impartial combinatorial game is equivalent to some form of 1-row Nim.

Corollary: If G is equivalent to $*n$ and H is equivalent to $*m$ then G+H is equivalent to $*(n \oplus m)$