



$E[pos]$  = expected winnings having reached position  $pos$

For final positions  $pos$ ,  $E[pos]$  = determined by rules

For non-final choice positions

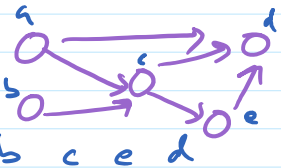
$$E[pos] = \max_{\text{choice } c} E[\text{next}(pos, c)]$$

position that results from making choice  $c$  at position  $pos$

For non-final random event positions

$$E[pos] = \sum_{\text{outcome } \sigma} P(\sigma) \cdot E[\text{next}(pos, \sigma)]$$

position resulting from outcome  $\sigma$  occurring at position  $pos$



(or any topological sort)

for every final position  $pos$   
 $E[pos] \leftarrow \text{payout}(pos)$

for every non-final position  $pos$  in order of  $\uparrow$  distance from end  
 if  $pos$  is a choice position

$max \leftarrow -\infty$

$argmax \leftarrow NIL$

for every choice  $c$

$e \leftarrow E[\text{next}(pos, c)]$

if  $e > max$

$max \leftarrow e$

$argmax \leftarrow c$

$E[pos] \leftarrow max$

$OPT[pos] \leftarrow argmax$

else

$e \leftarrow 0.0$

for every outcome  $\sigma$

$e \leftarrow e + P(\sigma) \cdot E[\text{next}(pos, \sigma)]$

$E[pos] \leftarrow e$

determined by rules

Coins: Start with  $n$  coins.

On each turn, flip as many of your remaining coins as you wish.

If  $\#T \geq \#H$ , lose all the  $T$

Else earn  $\#H$  points

Win at  $X$  points

Lose if no coins left and  $< X$  points

LOSE IF NO COINS LEFT AND  $>$  POINTS

Coin position = # coins left, # points left to target

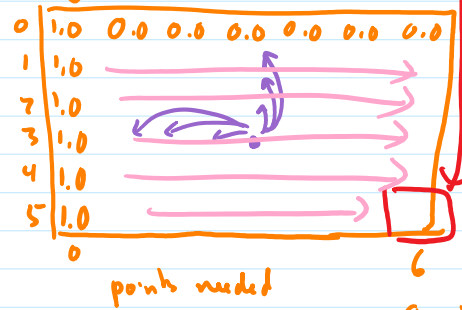
init row 0 to 0.0  
init col 0 to 1.0

```

for coins = 1 to startC
  for needed = 1 to target
    max ← -∞
    argmax ← null
    for flip = 1 to coins
      val ← 0.0
      for heads = 0 to flip
        if heads > flip - heads
          prob ← P(get heads H when flipping flip coins)
          val ← val + prob · E[coins][max(0, needed - heads)]
        else
          prob ← P(get heads H when flipping flip coins)
          val ← val + prob · E[coins - (flip - heads)][needed]
      if val > max
        max ← val
        argmax ← flip
    E[coins][needed] ← max
  
```

5 coins, play to 6  
start = (5, 6)

end = (x, 0) win  
or (0, y) loss



$\frac{\text{flip}}{\text{heads}}$   
 $2^{\text{flip}}$

anchor: positions at start of turn

component: positions during a single turn

number of anchors: every state of upper  
every state of FH, SS, LS  
every state of Y  
3K, 4K, C

$$\begin{array}{l} \text{unused, } 0, 1, 2, 3, 4, 5 \\ \downarrow \\ \underline{7} \leftarrow 6 \text{ categories} \\ \underline{3^3} \\ 15 \text{ unused, } 0, 50, 150, \dots, 1250 \\ \underline{28^3} \end{array}$$

$\approx 1$  trillion anchors  
+ 1600 pos/comp

$\approx 1.6$  quadrillion pos

$\approx 32$  years @ 1 million pos/sec

modification:  $E(\text{pos}) =$

For non-final choice positions

$$E(\text{pos}) = \max_{\text{choice } c} E[\text{next}(\text{pos}, c)]$$

For non-final random event positions

$$E(\text{pos}) = \sum_{\text{outcome } \sigma} P(\sigma) \cdot E[\text{next}(\text{pos}, \sigma)]$$

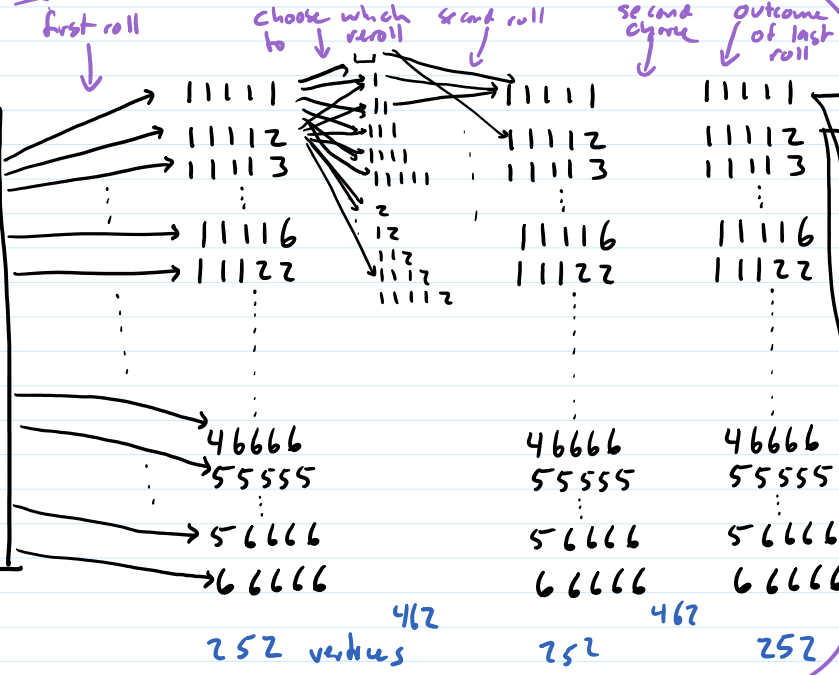
# anchors =

Yahtzee Graph

Component

|            |    |
|------------|----|
| Aces       | 1  |
| Deuces     | 2  |
| Trays      | 9  |
| Fours      | 12 |
| Fives      | 15 |
| Sixes      | 18 |
| 3 Kind     | 25 |
| 4 Kind     | 0  |
| Full House | 25 |
| S Straight | 30 |
| L Straight | -  |
| Chance     | 15 |
| Yahtzee    | -  |

anchor



|            |    |
|------------|----|
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|            |    |
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| Trays      | 9  |
| Fours      | 12 |
| Fives      | 15 |
| Sixes      | 18 |
| 3 Kind     | 25 |
| 4 Kind     | 0  |
| Full House | 25 |
| S Straight | 30 |
| L Straight | -  |
| Chance     | 15 |
| Yahtzee    | 60 |