Yahtzee

anchor: positions at start of turn

component: positions during a single turn

number of anchors: every state of upper
every state of FH, SS, LS
every state of Y
3K, 4K, C

unused, 0, 1, 2, 3, 4, 5
6 categories

7^6
3^3
15 unused, 0, 50, 150, ..., 1250
28^3

≈ 1 trillion anchors
· 1600 pos/comp

≈ 1.6 quadrillion pos

≈ 32 years @ 1 million pos/sec

modification: E[pos] = expected score on future turns starting from pos

For non-final choice positions

$$E[pos] = \max_{\text{choice } c} \left(E[next(pos, c)] + \underbrace{score(pos, c)}_{\text{score earned by making choice } c \text{ from position pos}} \right)$$

(only non-zero for end-of-turn pos in Yahtzee)

C —
Y —

For non-final random event positions

approx expected score when only Y left

score in chance 6 + 7
score in Yahtzee 0 + 21

$$E[pos] = \sum_{\text{outcome } \sigma} P(\sigma) \cdot \left(E[next(pos, \sigma)] + \underbrace{score(pos, \sigma)}_{\text{score earned when } \sigma \text{ happens in pos}} \right)$$

pos
always 0 in Yahtzee

anchors = categories used/unused (non-Y) 2^{12}
subtotal (0-63) 64
Yahtzee unused/0/50 3

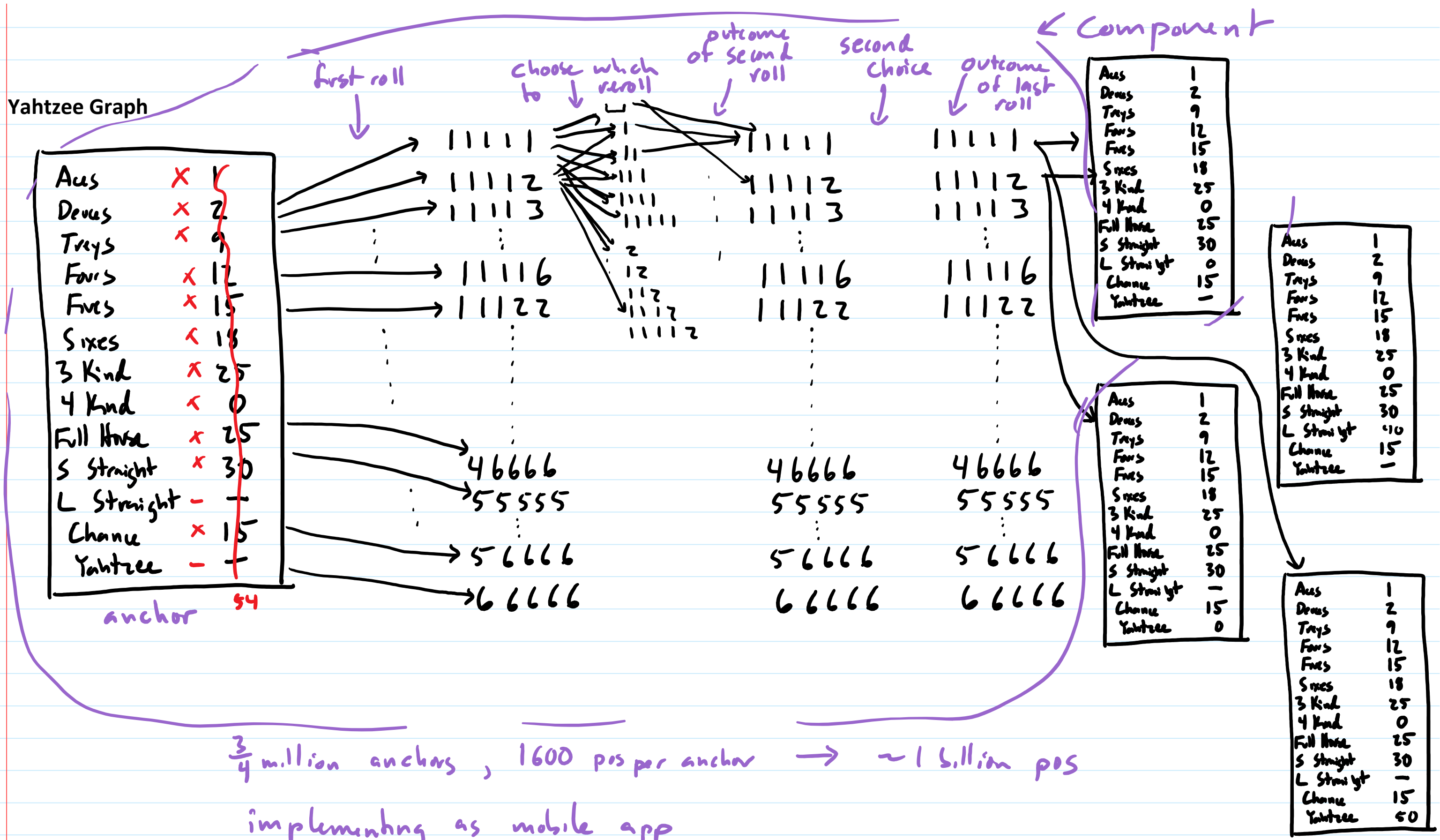
≈ $\frac{3}{4}$ million anchors
(some unreachable)

≈ 20 min @ 1 million pos/sec

1
2 —
3 —
4 —
5 —
6 —
subtotal 60

doesn't matter exactly what is in each, just that the sum is 60

Yahtzee Graph



$\frac{3}{4}$ million anchors, 1600 pos per anchor → ~1 billion pos

implementing as mobile app

only store values for anchors

reconstruct what's needed as game executes

↓

at pos, redo calc for that component

Two-player Zero-sum, probabilistic finite games

$E[pos]$ = expected wins for P1 winning from position pos
(= P(P1 wins) if no draws)

For P1 choice position

$$E[pos] = \max \quad E[next(pos, c)]$$

For P2 choice position

$$E[pos] = \cancel{\max}\ \min \quad E[next(pos, c)]$$

For non-final random event positions

$$E[pos] = \sum_{\substack{\text{outcome} \\ \sigma}} P(\sigma) \cdot E[next(pos, \sigma)]$$

2-player Yahtzee

optimal solitaire strategy is not optimal for 2-player

Aces	1	3	3K	15	20
Deuces	4	4	4K	0	
Treys	6	9	FH	25	0
Fours	16	12	SS	30	30
Fives	10	15	LS	0	40
Sixes	18	12	Chance		28
Bonus	0	0	Yahtzee		

Total

opt choice to maximize score is $\underline{0 \text{ in } Y}$ (0+21 vs 10+7)

opt choice to maximize wins $\underline{10 \text{ in } C}$ ($\underline{0.0}$ vs small but >0)

P(win) after 0 in Y

P(win) after 10 in C

anchors: Y 3 3^2
other cats 2 2^{24}
upper subs 64 64^2
score diff $\underline{3000}$

$\approx 2^{47}$ anchors

$\approx$ 100 trillion anchors

100 billion sec @ 1000 anchors/sec
$\approx$ 3000 years

2-player Yahtzee variant:

1) get score distribution of optimal solitaire player

2) compute strategy that maximizes the probability of beating the optimal solitaire player

Pig

2-players, turn-based

On each turn
- roll
- if 1, then turn over
- else add number to turn total
- decide: repeat
  - stop (and add turn total to score)

1^{st} to 100 points wins

$\underline{47}-36 \xrightarrow{\text{roll } 1} 47-\underline{36} \xrightarrow{\text{roll } 1} \underline{47}-36$

cycles → not finite :(

modify game: add turn limit (so now finite) — for high limit, opt strategy is very close to what it is in the no-limit game

$E[x, y, n]$ = expected # wins for P1 given score is x to y w/n turns left

expected # wins for P1 = P(P1 wins) + ½ P(draw)

n even: P1 turn
odd: P2 turn

T = score needed to win

$$E[x,y,n] = \begin{cases} 1.0 & \text{if } x \geq T \\ 0.0 & \text{if } y \geq T \\ 0.5 & \text{if } n = 0 \\ \max\limits_{2 \leq s \leq \max(2, T-x)} E[x,y,n-1] \cdot P_s(\text{total}=0) + \sum\limits_{k=s}^{s+5} E[\min(x+k, T), y, n-1] \cdot P_s(\text{total}=k) & \text{if } n \text{ even} \\ \min\limits_{2 \leq s \leq \max(2, T-y)} E[x,y,n-1] \cdot P_s(\text{total}=0) + \sum\limits_{k=s}^{s+5} E[x, \min(y+k, T), n-1] \cdot P_s(\text{total}=k) & \text{if } n \text{ is odd} \end{cases}$$

at start of turn, choose what to roll to

$P_s(\text{total}=0)$: prob of rolling a 1 before getting to s

$P_s(\text{total}=k)$: prob of ending at k when rolling to ≥ s