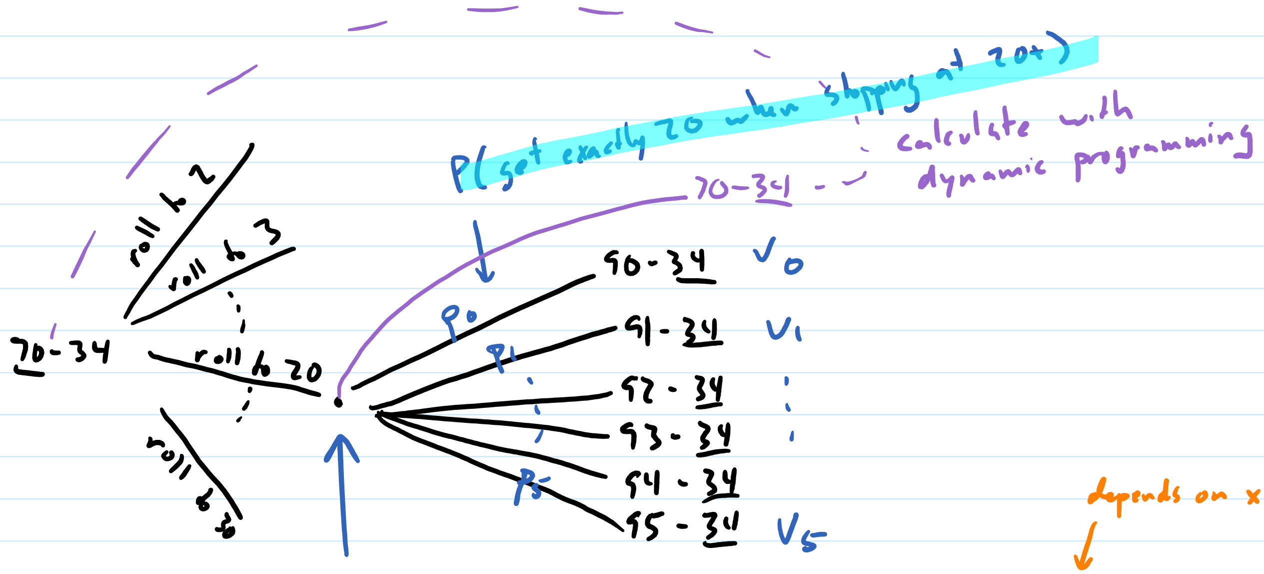
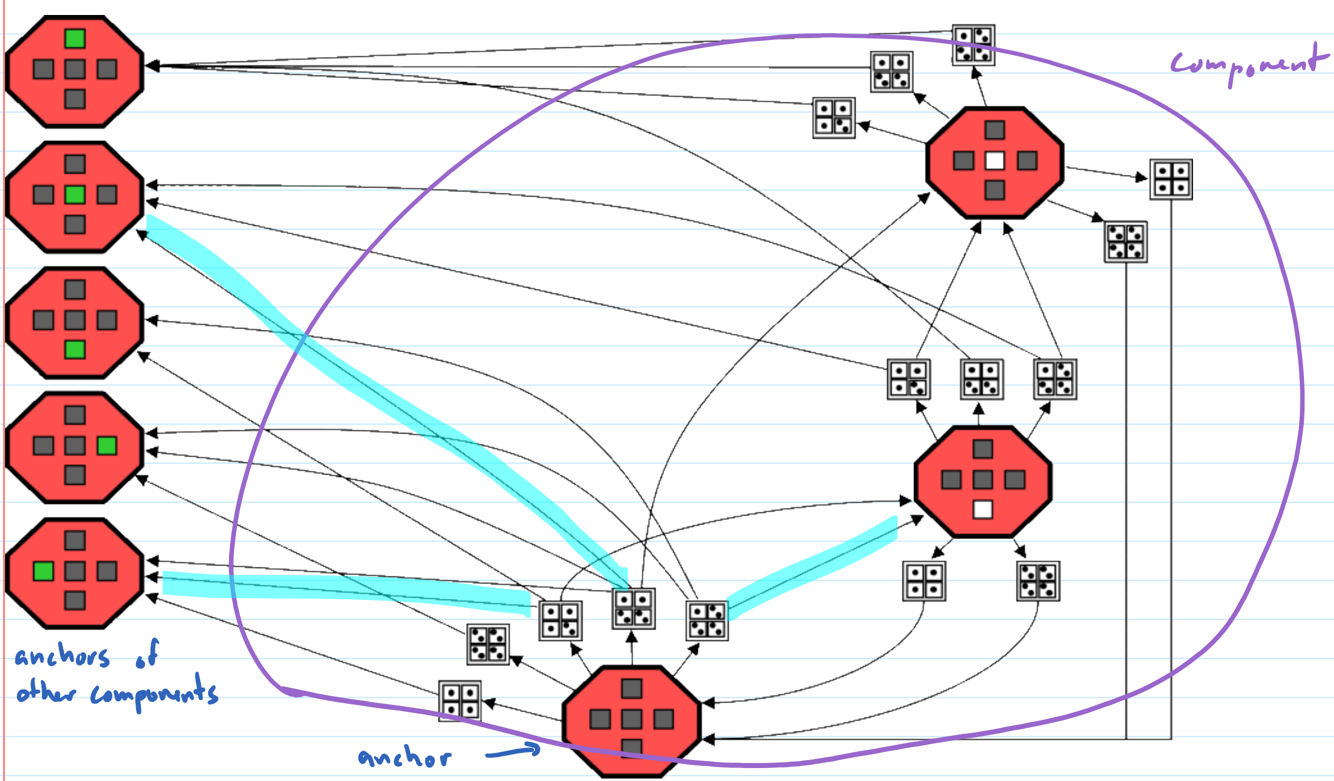
Fixed Point



$$\underbrace{x}_{\text{value of } \underline{70}\text{-}34} = V_0 \cdot p_0 + V_1 \cdot p_1 + \cdots + V_5 \cdot p_5 + \overbrace{V_{bust}}^{\text{depends on } x} \cdot P_{bust}$$

Can't Stop Graph



\# places marker can be in 2's column

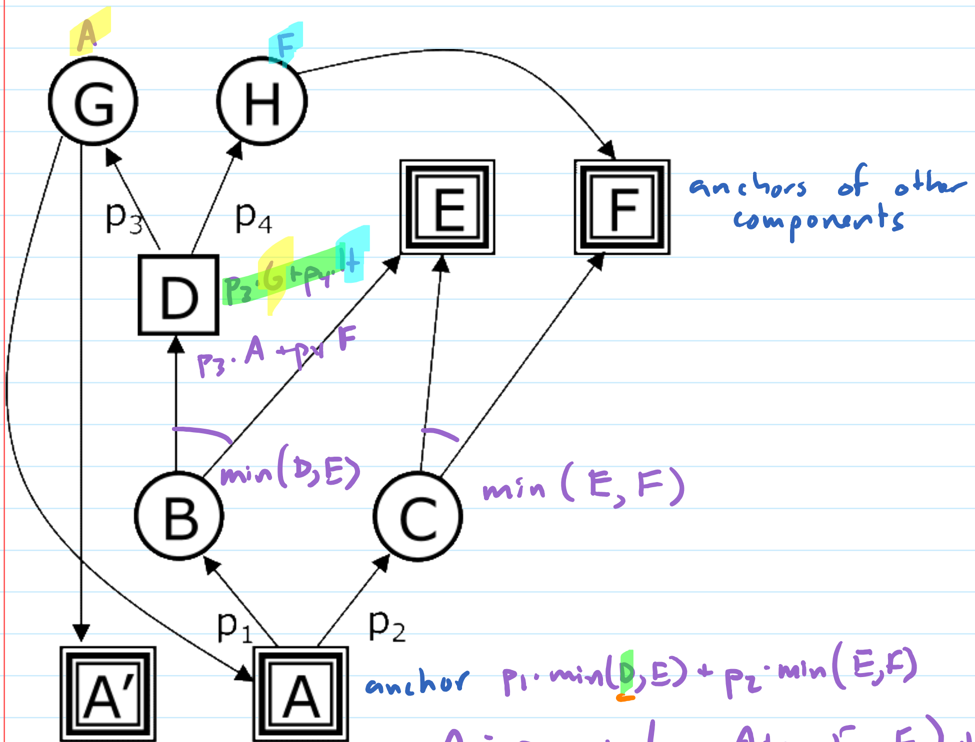
Number of anchors for standard solitaire game $\leq \underbrace{4}\cdot\underbrace{6}\cdot 8\cdot 10\cdot 12\cdot 14\cdot 12\cdot 10\cdot 8\cdot 6\cdot 4$

$\approx 7\frac{1}{2}$ trillion

\# places marker can be in 3's column

(not all are reachable, some are symmetric)

Piecewise Linear

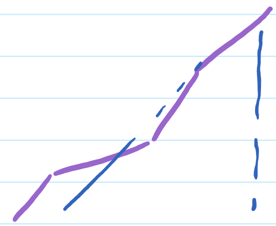


anchor $p_1 \cdot \min(D,E) + p_2 \cdot \min(E,F)$

$$A = p_1 \cdot \min(\underline{p_3 \cdot A + p_4 \cdot F}, E) + p_2 \cdot \min(E,F)$$

$$p_1 \cdot \min(\underline{p_3 \cdot A + p_4 \cdot F}, E) + p_2 \cdot \min(E,F) - A = 0$$

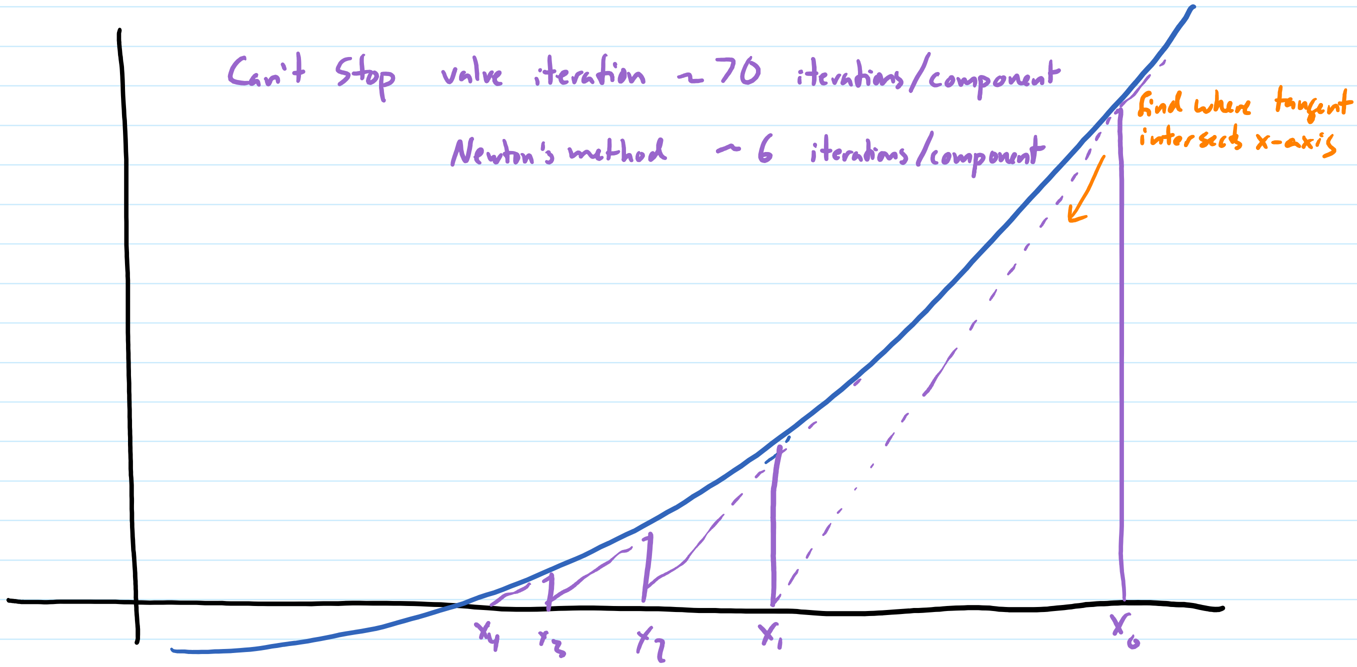
solve for A numerically



Newton's Method

Can't Stop value iteration ~ 70 iterations/component

Newton's method ~ 6 iterations/component



3+ Players

$E_1[pos]$ = expected wins at position pos for P1

$E_2[pos]$ = ⋮

⋮

$E_n[pos]$ = expected wins at position pos for Pn

For pos p, player i's turn, compute $E_i[pos]$ as usual (maximize)

for player j, $i \neq j$, once P_i's is known at pos, calculate $P(pos \to pos')$ using that strategy for successor positions pos',

$P(pos \to pos')$ → probability that P_i's strategy result in pos' when starting from pos

$$E_j[pos] = \sum_{pos'} P(pos \to pos') \cdot E_j[pos']$$

(value iteration, Newton's method must also be used differently when applicable for infinite games)