

# Sizes of Games

Minimax(pos)

If pos is <sup>game over</sup> terminal, return value as determined by rules  
 ↳ check who wins A: +1  
 B: -1  
 draw: 0

Else if pos is A's turn then return  $\max_{pos \rightarrow pos'} MM(pos')$

Else return  $\min_{pos \rightarrow pos'} MM(pos')$

avg branching factor (± moves at each position)

≤ 9 Tic-Tac-Toe

≤ 6 Mancala

2-player Tahtzee

≤ 48 Checkers

≤ 100 Chess

≤ 400 Go

avg length

very rough est on # pos  
 ≤  $3^9 \approx 20000$

≤  $\binom{49}{36} \approx 262 \text{ billion}$

≤  $(2 \cdot 10^5)^2 = 4 \cdot 10^{10}$

≤  $\binom{32}{12} \binom{20}{12} \cdot 2^{12} \cdot 2^{12} + \binom{32}{11} \binom{21}{12} \cdot 2^{11} \cdot 2^{12}$

≤  $\binom{64}{8} \binom{56}{8} \binom{48}{2} \binom{46}{2} \binom{44}{2} \binom{42}{2} \binom{40}{2} \binom{38}{2} \cdot 36 \cdot 35 \cdot 34 \cdot 33 + \dots$

≤  $3^{19 \cdot 19} \approx 10^{172}$



[http://en.wikipedia.org/wiki/Game\\_complexity](http://en.wikipedia.org/wiki/Game_complexity)

<http://xkcd.com/1002/>

What to do with games of high complexity?

heuristics - estimate of position value

Simple Ex : checkers % of remaining pieces that are black (scaled to -1, 1)

chess assign value to each type of piece (pawn=1, ..., queen=9)  
 % of total value belonging to white (scaled to -1, 1)

heuristic fun based on how deep recursion can go

Minimax(pos, h, depth)

If pos is terminal, return value(pos)

If depth = 0, return h(pos)

Else if pos is A's turn then return  $\max_{pos \rightarrow pos'} MM(pos', h, depth - 1)$

else if pos is terminal return value(pos)

else return  $\min_{pos \rightarrow pos'} MM(pos', h, depth - 1)$

Negamax(pos, h, depth, sign)

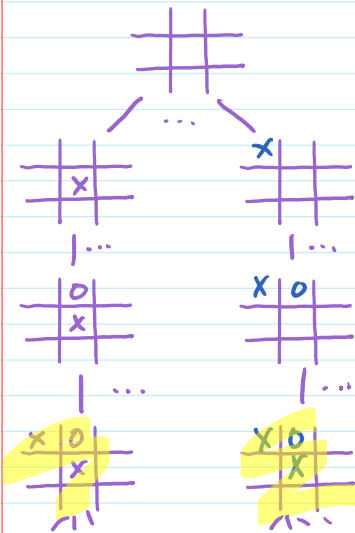
If pos is terminal, return  $value(pos) \cdot sign$

If depth = 0, return  $h(pos) \cdot sign$

return  $\max_{pos \rightarrow pos'} -MM(pos', h, depth - 1, -sign)$

Iterative Deepening - to allow a response after a set time  
 $depth \leftarrow 2$   
 while not out of time  
 do  $MM(pos, h, depth)$   
 $depth \leftarrow depth + 1$   
 return last move for last call that finished

Transposition Table - memo



same pos on multiple branches

If pos is terminal, return  $value(pos)$

If depth = 0, return  $h(pos)$

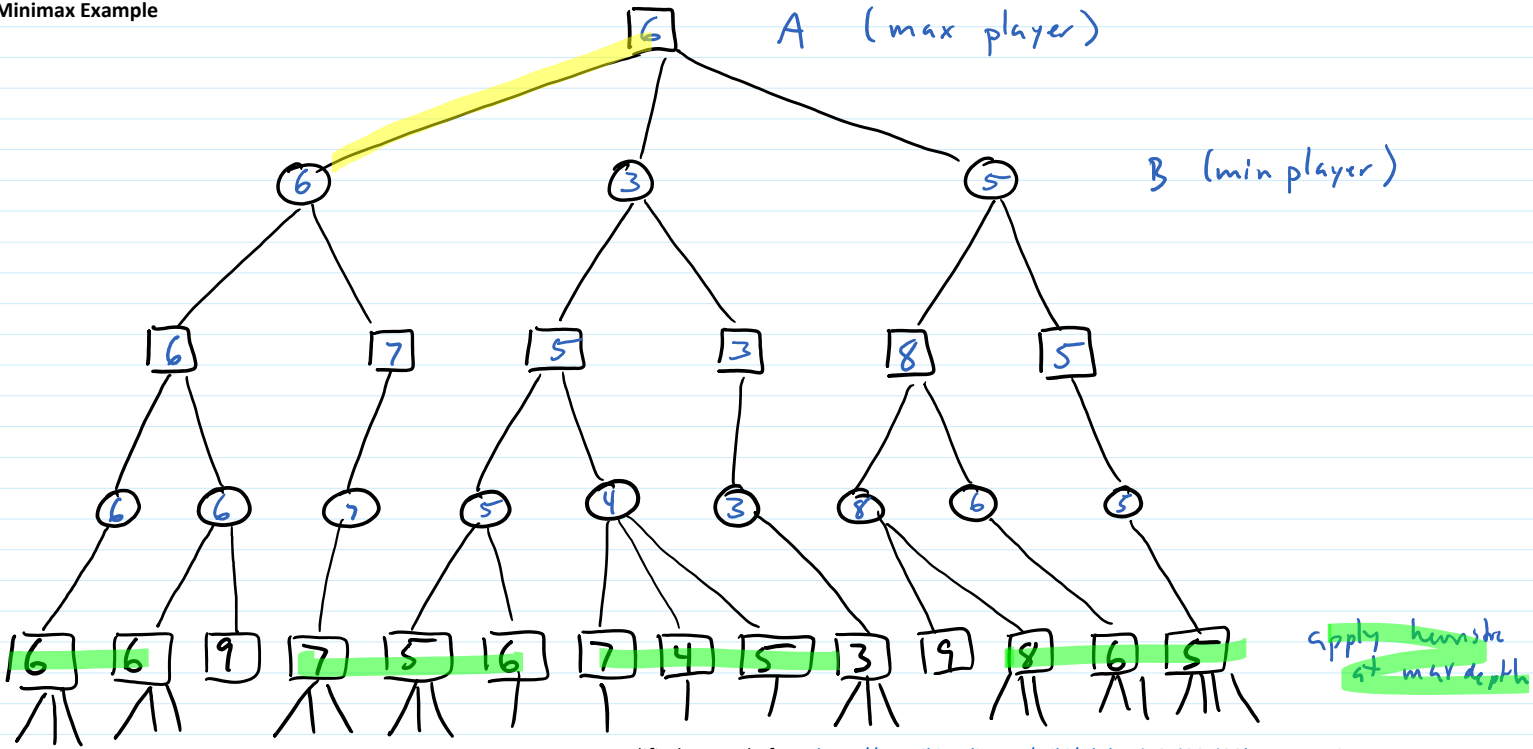
If pos in TT return  $TT[pos]$

Else if pos is A's turn then  $TT[pos] = \max_{pos \rightarrow pos'} MM(pos', h, depth - 1)$   
 return  $TT[pos]$

Else  $TT[pos] = \min_{pos \rightarrow pos'} MM(pos', h, depth - 1)$   
 return  $TT[pos]$

(memory intensive)  
 $\hookrightarrow$  replacement policy?

Minimax Example



Modified example from [http://en.wikipedia.org/wiki/Alpha%E2%80%93beta\\_pruning](http://en.wikipedia.org/wiki/Alpha%E2%80%93beta_pruning)