A Computational Tool for Assessing the Risk of Collateralized Loans

Abstract

Poorly managed risk of collateralized loans partially caused Iceland’s recent financial crisis and this area of finance is lacking adequate methodology and tools. We adapt the traditional value at risk (VAR) measure to account for the margin of a loan’s collateralization and experimentally determine thresholds such that we should accept a loan proposal if its VAR is below the threshold and reject it otherwise. We generate random data on which we test our decision rule and find that it is nearly always more profitable than accepting all loan offers. The mean difference in returns is $412,030 with a standard deviation of $289,350 across all four dates, but it is difficult to generalize these results without real data.

Problem

Iceland fared particularly poorly in the financial crisis of the last two years and this was largely caused by a fundamental flaw in the lending policies of many Icelandic banks: though each lending desk within a given bank tended to follow proper diversification guidelines when issuing a collateralized loan, aggregating loan sheets across the entire bank revealed that a large fraction of the bank’s loans were backed by the same or highly-correlated assets. This meant that the risk of each collateralized loan was much higher than the banks realized, so that when liquidity dried up and the value of these assets fell, the terms of the loan required banks to post a margin call, asking lenders to put up additional collateral. This was achieved by offering more of the same asset, defaulting on the loan, or selling the initial collateralized asset to pay off some of the loan. The first and second options left banks with more of an asset whose value was declining and the third option further depressed the asset’s price and affected the margins on other loans. Under these conditions, banks often chose not to ask borrower for additional collateral, resulting in even riskier positions.

On the simplest level, this is an issue of communication and coordination between the various desks within a bank, but considering that each loan has a different length, risk of default, and probability of prepayment, the problem becomes highly dynamic. If banks had employed a methodology to account for the correlations between collateral assets, they may have avoided such heavy losses. We developed a proof-of-concept computational tool to assess the risk of a bank’s collateralized loans and help a loan officer decide which new loans to accept. Our approach is innovative in examining collateral as a source of risk. There is growing demand for collateral management tools by both banks and regulators who wish to assess a bank’s financial stability (The Economist 2010).

Devising a Decision Rule

We wanted to develop a tool that will help loan officers decide when to accept a proposed loan. The question can be restated as whether the bank would buy the collateral asset as an investment if it were not collateralizing a loan. Then we can think of the lending book as a portfolio of investments and apply the principles of diversification. We avoid assets that are highly positively correlated with many of the assets already in our portfolio because they will magnify losses and we seek assets that are negatively or uncorrelated with most of our existing assets since they are good hedges. The optimal decision is less obvious when collateral assets are
slightly correlated with our portfolio – we may be willing to accept some of these loans if their return is worth the risk. We should also note that the sequence of proposed loan matters because the collateral of one accepted proposal is immediately added to the lending book and its correlations affect the evaluation of the next proposal.

In addition to risk, there are two important factors that affect a bank’s decision to accept a loan contract. Each loan has an interest rate that determines the amount the bank will earn on the loan. Loans are initially overcollateralized at a margin above loan’s value so that the bank is guaranteed a buffer against the price volatility of the collateral assets. Banks may choose to increase margins when markets are more volatile. While a higher interest rate results in higher returns for the bank, a higher initial margin results in lower risk. However, as (Stiglitz and Weiss 1981) explain, increasing rates and margins could attract riskier borrowers since the more credit-worthy can obtain loans with better terms. To avoid these complications, we set the interest rate exogenously and hold it constant over time. Likewise, each loan begins with the same initial margin.

Ideally we would like a pair of thresholds, one cumulative and the other incremental, such that we will not accept a proposed loan if the total risk of the bank’s collateral assets exceeds the cumulative threshold or if the additional risk of the proposed loan exceeds the incremental threshold. But first we need reliable measure of risk.

**Value at Risk**

Value at risk (VAR) is a statistical measure of downside risk based on current positions. It summarizes risk in a single, easy-to-understand number: VAR is the worst loss on a portfolio of assets over a target horizon such that there is a low, prespecified probability that the actual loss will be larger. Define \( c \) as the confidence level and \( L \) as the loss, expressed as a positive number. VAR is the smallest loss, in absolute value, such that

\[ P(L > VAR) \leq 1 - c \]

(Jorion 2007, 105-106). Now that we have defined VAR, we need a way to compute it for a portfolio of assets. Let \( x \) be the vector of dollar exposures to each asset and let \( \Sigma \) be the matrix of covariances between the assets so that \( x' \Sigma x \) is the variance of the portfolio rate of return in dollar terms. If \( c \) is the confidence level and \( \alpha \) is a standard normal deviate such that the probability of observing a loss worse than \(-\alpha\) is \( c \), then portfolio VAR is

\[ Portfolio\ VAR = VAR_p = \alpha \sqrt{x' \Sigma x} \]

(1) 

Incremental VAR, which is the change in VAR due to a new position, is

\[ Incremental\ VAR = VAR_{p+a} - VAR_p \]

(2) 

(Jorion 2007, 162-168).

In addition to the portfolio of collateral assets, we must specify our confidence level \( c \) and time horizon. The choice of these values depends on the data and how we intend to use the VAR measures. More data means more extreme values, allowing for a higher confidence level. But a higher confidence level also means a larger VAR measure since the event of loss is less common. The horizon is often based on the amount of time it takes to liquidate the assets in the portfolio. The Basel Committee imposes a 99% confidence level over a 10-business-day horizon when computing VAR (which is then multiplied by 3) to determine banks’ minimum capital requirement for a portfolio of marketable assets (Jorion 2007, 120). We have daily price data and we want to estimate regular losses in the lending book, so we chose a 95% confidence level and
a 1-business-day horizon. This means that the collateral assets will experience losses at least as great as our VAR measure about 12 times per year.

**Extending VAR**

The standard equation for portfolio VAR does not take into account the degree to which assets in the lending book are overcollateralized, which is a significant shortcoming if we plan to use VAR as our basis for risk assessment of collateralized loans. We would like to include only the risk that losses exceed the buffer of each loan’s margin. We can see in the portfolio VAR equation that the greater the dollar exposures in the vector \( x \), the higher the VAR measure, so we want to decrease the dollar exposure of a loan proportional to its margin. Thus, we divide the value of each loan by its margin so that the exposures in \( x \) become the values of the loans minus the dollar amount of the overcollateralization. This margin-weighting improves the ordering of our VAR measure. For example if there are two loans of equal amounts that are collateralized by the same asset but one has a margin of 110% and the other has a margin of 120%, then the one with a 120% margin will result in a lower VAR measure.

**Evaluating the Decision Rule**

Now that we have a risk-assessment framework for collateralized loans, we need to implement and test it. We want to know how much better our lending book will perform if it follows our decision rule. Given an evaluation date, we generate a lending book (B) with random loan amounts, random stocks as collateral, and random termination dates beyond the evaluation date. Next we generate another series of random loan proposals beginning on the evaluation date and, considering each proposal in turn, we compute the margin-weighted portfolio VAR before and after adding it to the lending book. We keep two copies of the original lending book (B): one, named (T), that includes only those loan proposals that meet the criteria of the decision rule and another, called (C), that includes all proposed loans. Since the total amount of the loans in (C) will be at least as large as that in (T), we scale the amount of all accepted loans in (C) so that the sum of the loan amounts in each book is identical. Then we look ahead to calculate the margin of each loan on its termination date. If this margin is greater than 100%, we compute returns as the present value of the interest on the loan amount, paid on the date of the loan’s termination. If the margin is less than 100%, that loan represents a loss, so we compute the present value of the collateral assets, liquidated on the loan’s termination date. Finally, we compare the returns from the lending book (T) generated by the decision rule to the one (C) that contains all assets. Next we build our tool.

**Implementing the Decision Rule**

Though we considered coding the tool in Python to leverage its prewritten packages for designing user interfaces, we ultimately chose MATLAB for its rapid prototyping capabilities and native matrix operations. It also has a decent GUI editor, which shortened the time necessary to get a user interface running. Manipulating MATLAB data structures is not particularly speedy, which impacts the running time of the tool.

The original lending book (B) contains 100 loans and the amount of each is equal to the absolute value of a random variable from a standard normal distribution multiplied by $1,000,000, so that the average loan amount is about $800,000 and the average lending book is worth about $80 million (see Figure 1 for an example distribution of amounts). The length of each loan is a uniform sample from the set 12 months, 18 months, and 24 months since
borrowers can find shorter-term financing in the futures market. All loans are collateralized by a single stock drawn uniformly from a subset of 36 stocks in the S&P 500 index (explained in more detail below), meaning that the same stock on average collateralizes about 3 loans. The start date is drawn uniformly from the range of dates within the loan’s length prior to the evaluation date so that all loans in the lending book are still open on the evaluation date. We compute the initial number of shares by dividing the loan amount by the stock price on the loan’s start date, multiplied by the initial margin of 140% (see Figure 2 for a example distribution of shares). We then update the margin on the evaluation date by multiplying the initial number of shares by the price of the stock on the evaluation date divided by the amount of the loan (see Figure 3 for an example distribution of margins). The sequence of loan proposals consists of 25 new loans generated in the same way as those in the initial lending book but all starting on the evaluation date.

To compute VAR, we need a covariance matrix, and we can get the most realistic sense of the correlations between stocks if we base ours on historical stock price data. I downloaded a list of the constituent stocks in the S&P 500 index from the Standard and Poor’s website. I then modified a MATLAB function I found online to retrieve from the Yahoo! Finance website the adjusted close prices for all of the S&P 500 stocks. We cannot estimate the effects of a liquidity crisis without at least one liquidity crisis in our data, so we want data from a long time period. Fortunately, we are also happy with a small pool of stocks, since this will increase the concentration of correlated loans in our lending book. Thus, I restrict the price data to those stocks with a 30-year window ending on March 24, 2010 and eliminate tickers with dates that are inconsistent with GE’s, which leaves us with 36 tickers. This is not a particularly representative sample because they are all large cap stocks and there is a survivorship bias among them since they have all remained in the index for over 30 years. We cannot include prices past the evaluation date, so if we need about 10 years of data for a reliable covariance matrix, then we can simulate any date between March 24, 1990 and March 24, 2010.

To verify the accuracy of our VAR computation, I looked at the portvrisk.m function in MATLAB’s Financial Toolbox and confirmed it is identical to the code I wrote myself. I also tested my code on an example in (Murphy 2008) and it produced the same results. The difficulty was in finding reasonable cumulative and incremental thresholds to use in our decision rule. We considered setting the cumulative threshold as some fraction of the bank’s equity, but we failed to find a good cutoff, especially since our margin-weighted VAR measure cannot be interpreted the same way as traditional VAR. We are able to determine a good incremental threshold experimentally by maximizing the difference between returns in lending books (T) and (C) over a range of threshold values.
When computing the return on the lending books (T) and (C), we assumed a constant real annual interest rate of 5%. (We leave inflation out of the simulation.) All loans are evaluated on their termination date only, which prevents us from having to check the margin each day and from trying to predict the probability of meeting the margin call and the risk of default. Let $R_t$ be the return at the loan’s termination date, $i$ be the annual interest rate of 5%, and $t$ be the length of time between the evaluation date and the termination date in years. Then we express the present value of returns $R_0$, discounted in terms of the evaluation date, as

$$R_0 = \frac{R_t}{(1+i)^t}. \tag{3}$$

**Results**

Our first objective was to determine experimentally the optimal incremental VAR threshold below which we should accept loan proposals and above which we should reject them. Since it is likely that the threshold varies depending on the evaluation date, I choose four evenly distributed dates: March 24, 1990; March 24, 1995; March 24, 2000; and March 24, 2005 (it is possible that the behavior of stock prices in March is not representative of the rest of the year). This allows for at least 10 years of price data to compute the covariance matrix as well as more than a 2-year buffer to ensure all loan termination dates are within the range of our price data. I then ran the simulator on each of these dates, generating 10 lending books for the threshold values between $0$ and $40$ million in increments of $100,000$. In Figures 4 through 7, I plot the mean difference in return between the lending book (T) generated by the decision rule and the one (C) that includes all assets. These curves have been smoothed by a weighted sum of the nearest 5 values.

![Figure 4](image1.png) **Figure 4** Mean difference of returns on 10 lending books for each threshold value between $0$ and $40$ million in $100,000$ increments, evaluated in 1990

![Figure 5](image2.png) **Figure 5** Mean difference of returns on 10 lending books for each threshold value between $0$ and $40$ million in $100,000$ increments, evaluated in 1995

In 1990 and 1995, the mean benefit rises sharply at low threshold values and decreases similarly rapidly so that it is effectively $0$ past the $20$ million threshold. In 2000 and 2005, however, the mean benefit increases more gradually and becomes noisy at the peak. The decline in mean payoff is even slower, not even reaching $0$ by the $40$ million threshold.
To get a better estimate of the optimal thresholds for each of these dates, I evaluated 100 lending books for each threshold within a narrower range that contained the maximum, again in $100,000 increments. Figure 8 shows the mean difference in returns in 1990 of this higher-resolution simulation, where the threshold values are between $0 and $6 million. Figure 9 does the same but for the threshold range from $2 to $6 million in 1995. Figure 10 spans the $5 to $20 million range for 2000 and Figure 11 goes from $5 to $25 million in 2005. These curves have also been smoothed by a weighted sum of the nearest 5 values.

We can see the maximum threshold more clearly in these plots than we could in Figures 4 through 7. I computed the argmax of each curve and recorded this value as the optimum threshold. For 1990, it was $2 million, for 1995 it was $3.4 million, for 2000 it was $13.6 million, and for 2005 it was $16.6 million.
Finally, I ran the simulator on all four dates with their corresponding optimal thresholds. I generated 1000 lending books for each date-threshold pair and calculated the differences in returns. These principal results are presented in Figures 12 through 15.

We observe a mean increase of $381,150 with a standard deviation of $205,860 in 1990, a mean increase of $311,390 with a standard deviation of $68,657 in 1995, a mean increase of $573,130 with a standard deviation of $426,850 in 2000, and a mean increase of $382,460 with a standard deviation of $260,600 in 2005. In 1990 and 1995, the differences in returns were not less than $0, but in 2000 0.4% were below $0 and in 2005 0.01% were below $0. Thus we find in nearly all cases that our decision rule performs better than no selection mechanism.
It is clear that the evaluation date has a significant impact on both the optimal threshold value and the distribution of returns. The threshold increases significantly over time, with an especially large jump from 1995 to 2000 that cannot be explained by inflation. Perhaps the stocks in our pool became more highly correlated over our evaluation window. This variability would be a problem if we were trying to determine an optimal threshold in real time, without the complete information of hindsight, since we would be seeking a moving target. The distributions of returns shift towards lower returns and get tighter over time. The economic slump following 9/11 could explain some of the anomalous properties of the 2000 sample.

Implementing the Interface

I also implemented a GUI interface as a prototype of the tool a loan officer might use. It functions the same as a single iteration of the simulation, but allows the user to enter the loan parameters and make the final decision to accept or reject the proposal. The GUI is invoked by entering [interface ‘yyyy-mm-dd’] on the MATLAB command line, where yyyy-mm-dd is the evaluation date between March 24, 1990 and March 24, 2008. After choosing a loan amount, length, and collateral stock, click the Submit button to see the incremental VAR of the proposed loan. If the VAR measure is green, the decision rule recommends accepting the loan and if it is red, then the decision rule suggests rejecting it. Click the Accept or Reject button to record the decision. Then either propose more loans or, if at least one loan has been accepted, click the Simulate button to see the results. The difference in returns between the lending book of accepted loans and the lending book in which all proposed loans are included is displayed on the MATLAB command line once the GUI window closes. This implementation allows the user to experiment with decision rule in an interactive environment.

Unresolved Problems

The assumptions and generalizations we made leave several significant problems unresolved. Many of these problems arise because we are working with a simulation rather than real-world data. We must make abstractions, but we must also be wary of how they affect our results. First, due to our position-weighting method, the VAR measure loses its intuitive interpretation. This is acceptable if we only use it as a benchmark, but its value is not very informative to a bank’s risk manager. Second, our VAR measure assumes the margin’s effect on
risk is linear. That is, we treat a margin increase from 120% to 130% the same as an increase from 100% to 110%. There are flaws in our evaluation methodology as well. Our decision to scale all of the new loans in lending book (C) is necessary to ensure the potential returns of each lending book is identical, but it results in a comparison between fewer large loans in (T) and more small loans in (C). Some proposed loans are so obviously bad that they should not be included in (C), but our approach ignores this fact. We could improve the quality (but also increase the complexity) of our simulation by looking at daily price changes instead of only evaluating loans on their termination dates. This would let us include margin calls and force us to account for default risk. Including more stocks, allowing different (and multiple) assets as collateral, varying the size of the lending books, and introducing the reputation of borrowers would also make the tool more realistic.

Conclusion

The need for reliable risk assessment methodology for collateralized loans is growing and there is considerable demand for collateral management tools. This project introduces the margin-weighting extension to traditional VAR and demonstrates the feasibility and profitability of using such a measure to decide when to accept and reject loan proposals. But, as former MIT AI Lab director Rodney Brooks reminds us, “Simulations are doomed to succeed.” We made many assumptions in the design and implementation of the tool that could have undermined the soundness of our results. To thoroughly evaluate the effectiveness of this tool, we need real rather than artificially generated data. If we could apply the same decision-making system to a real bank’s lending book and proposed loans, then we could record its hypothetical returns over time and collect enough empirical evidence to prove the robustness of this tool.

References


