Synchronization

Siddhartha Banerjee advised by Prof. Kumpati S. Narendra, and Prof. Steven W. Zucker

The project plans to explore the problem of synchronization among multiple agents in a variety of situations. A lot of work by distinguished researchers has already been done to explore this problem; indeed a lot of models and methods of analyzing synchronous behavior have been proposed. This project will first try to understand the gist of the proposed models and methods. Then, through a series of simulations and analyses, the project will try to attempt an explanation of new phenomenon through older models.

Why Synchronization?

Synchronization is a broad and ubiquitous problem that has been on the minds of human researchers for hundreds of years. Understanding the nature of synchrony has occupied the best of human minds, and applications of the consequences of synchrony amaze and delight us even today. Indeed, the understanding and use that humans have gained from synchronization in nature have had far-reaching consequences in every field of human endeavours.

The most obvious benefit of Synchrony (or “Sync” as Steven Strogatz likes to call it\(^1\)) has been in the field of Engineering. While early engineers dealt with the problem(s) of synchronizing mechanical devices accurately to the cycle of days, their present day peers strive to develop synchronized time-keeping mechanisms in electronic circuits to ensure effective communication between computers. In fact, this effective communication between computers (based on Sync) is the foundation for modern day multi-processor systems, the internet (a distributed system), global positioning systems, and much more.

Aside from Engineering, surprising amount of progress and understanding of Sync has come from other fields of human research as well. In the realm of Biology, the flashing of fireflies and the steady electric pulses powering the heart have all been equated to the synchronization equations seen in Engineering. Indeed, research shows that even basic muscular contractions in vertebrates have their origins in synchrony between neurons in an organism. Similarly, the Social Sciences have tried to explain sync through the analysis of networks and crowds. Their models on the propagation of synchrony through both sparse and dense networks have led to new insights into the propagation of diseases and outbreaks. Finally, and most importantly, the field of Physics has come up with discoveries of Sync in the natural world that have questioned and advanced our fundamental beliefs in the working of nature. In fact, discoveries like that of Quantum Synchrony in Physics has led to the creation of the Josephson Junction, which in turn has led to remarkable inventions such as MAGLEV trains, MRI machines, etc.

Although Sync has proven to be one of the most useful, ubiquitous, and even sometimes the most natural phenomenon, it is perhaps one of the least understood. Decades (and

---

sometimes even centuries) of research\(^2\) have yet to come up with accurate predictions for all kinds of synchrony. Indeed, as will be elaborated on later, there are forms of synchrony that remain a complete mystery.

Previous work done on Synchrony has come from researchers such as Kuramoto, Kopell, and Winfree. However, because these researchers have each tackled the problem from the perspective of multiple diverse fields, their conclusions have sometimes been confusing and disjointed. Hence, this project hopes to first conclusively understand previous work done on Sync before trying to provide solutions to new problems of synchrony.

**Objective 1: Previous Work done on Sync**

As mentioned earlier, there has been a vast amount of work that has been done on Sync. The following paragraphs contain a brief overview of some of these works.

Van Der Pol was the first researcher to contribute meaningfully towards Sync. His work on Limit Cycles, and their corresponding stable and unstable states was the first step towards the characterization of a system in oscillation. Limit cycles, rather than states, are the fundamental descriptive entities of oscillating systems; and the Van Der Pol oscillator given by

\[
x'' + \epsilon(x^2 - 1)x' + x = 0
\]

is one of the most notable examples of an oscillator with a “stable” limit cycle.

Wiener, however, was the first put forth theories on collective synchrony. Although he could not find appropriate mathematical expressions to characterize his notions of synchrony, his notions were verified by Winfree who put forth the following expression for a coupled population of oscillators:

\[
\dot{\theta}_i = \omega_i + \left( \sum_{j=1}^{N} X(\theta_j) \right) Z(\theta_i) \quad i = 1, 2, \ldots, N
\]

where \(\theta_i\) and \(\omega_i\) are the phase and natural frequency of the \(i\)th oscillator, while the other terms signify the coupling.

Kuramoto then built on Winfree’s work to produce a more mathematically tractable expression for coupled oscillators:

\[
\dot{\theta}_i = \omega_i + \sum_{j=1}^{N} \Gamma_{ij}(\theta_j - \theta_i) \quad i = 1, 2, \ldots, N
\]

where \(\Gamma_{ij}\) now signifies the coupling between oscillators.

Finally, contemporary mathematicians such as Strogatz and Kopell have built on the work of Kuramoto and his predecessors. While Strogatz has contributed to the field of synchrony

through studies of chaos in chemical reactions, Kopell has made significant breakthroughs in the design and analysis of Pattern Generators.

In order to understand this vast amount of work that has gone into Sync, the project will try to recreate all these models and judge their applicability to real world problems. Beginning with basic studies of entrainment of Van Der Pol oscillators to sinusoidal and other relaxation oscillators, and continuing on to studies of infinite oscillators forming patterns, this project will attempt to understand the gist of the different models of synchrony that have been proposed and try to place the results in a uniform framework.

Objective 2: Apply Sync to New Problems

Having gained a firm grasp of the intricacies of synchrony suggested by previous models, the project will then attempt to explain two of the following phenomenon that have yet to be explained through synchrony:

1. In the presence of predators, birds flying in flocks have been observed to form straight line patterns.
2. A school of fish in the presence of a predator synchronize themselves into toroidal shapes.

Proposed Deliverables

This Proposal aims for 2 main results by the end of the project:

1. An understanding, survey and possible amalgamation of Sync through the simulation and analysis of various models applied in different contexts.
2. A set of new formulations of Synchrony in the context of the new problems mentioned.

The proposed deliverables of this project are:

1. A Research paper.
2. Code for Simulations run and tests made. This code will most likely be in the form of MATLAB scripts.
3. A final project presentation.

---