Traveling Spacemen: Investigating TSP with Moving-Bodies

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The Traveling Salesmen Problem (TSP) is regularly regarded as one of the classic challenges in computer science. Much previous literature, and indeed previous Yale classwork, on the problem traditionally focuses on the static case. That is, a situation where the challenge is to visit each of a list of locations only once with minimal distance. In many cases this is presented as a graph – with each node representing a location and the edge weights between nodes representing distance. Despite its challenges, much is known about the classic TSP such as its inclusion in NP-Hard classification, various dynamic programming solutions, etc.

Unfortunately, things in the real world often move. This becomes even more increasingly true in our interconnected and mobile computing world. Therefore, being able to have a good understanding of the TSP in the case of moving bodies rather than set locations is an important concern. In truth, the classic TSP is really just a special case of the moving bodies TSP – one with all bodies’ velocity equal to zero. Spending a semester digging into the additional parameters and concerns that present themselves once locations are set in motion will obviously not bring final answers, but it should raise attention to the right types of questions and be able to get a glimpse of some more sophisticated solution algorithms and analysis of performance and complexity.

Figure 1 - An example (a) of a moving-body TSP problem and (b) its optimal solution. (from Helvig 2003)

Although early in investigations, there are a few of the differences I am specifically interested in pursuing further: (1) the emergent metric choices of time or distance, (2) alterations in currently understood dynamic programming algorithms that solve the static TSP, and (3) NP-Hard classification.

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1 CS 365 – one of the foundational courses for the major - covers TSP in the static case
Time vs. Distance
In the static TSP problem, we are finding the best solution as function of both distance and time simultaneously. Indeed, to the static salesman distance is time. This is not true for the moving-body TSP. There emerges a choice of minimizing distance traveled by the salesman, or time for the salesmen to complete the trip. These are very separate questions, as can be illustrated by the following example: a traveling spaceman who wishes to touch all the planets before returning to earth may minimize his total distance traveled by simply waiting for a full planetary alignment and then simply hop each planet in order before returning home. Suppose this only happens once every hundred thousand years. Surely, though this yields the shortest possible path through the solar system, it also would not be the fastest given a spaceship of even modest velocity. This was a distinction that we didn’t have to deal with in the static TSP, and it will be interesting what types of challenges or changes this could bring forth.

Algorithms to Solve the Moving-Body TSP
I plan to build working implementations of various possible moving-body TSP algorithms, and compare performance to each other and the brute force algorithm. Also, I will look into approximation algorithms for the static TSP and see how easily they will apply to the non-static case. Dynamic programming algorithms have already proven themselves to be strong tools for a reasonably fast TSP solver, but many of the popular strategies may fail to apply to the non-static case. I will dig deep into the dynamic programming approaches that currently exist, and work to apply them to the moving-body TSP.

NP-Hard Analysis
As mentioned previously, the TSP is a classic computational challenge and is well known to be an NP-hard problem. How about the moving body TSP? Does the fact that your destinations have a non-zero velocity change this? What about if the bodies’ movement is non-zero in both velocity and acceleration – in effect that they are moving at varying speed dependent on time? Are we still dealing with a provably NP-Hard problem?

To answer these questions, I will attempt to develop a proof demonstrating that the moving body TSP is solvable with polynomial calls to one of the other known NP-Hard problems.

Summary
The moving-body case is non-trivially different from the static case that is often the focus of TSP research. Through this project, I not only hope to gain a stronger understanding of the best TSP algorithms that currently exist for the static case, but also to look into previous attempts to solve the moving-body case and even create and test approaches of my own. I hope to graduate knowing that if I left planet earth right now on a spaceship with known velocity, what would be the fastest way to touch every planet and return home?
Deliverables

- Research what has already been done
  - Publications
    - Applegate et al., *The Traveling Salesman Problem*\(^2\)
    - Helvig, *The moving-target traveling salesman problem*\(^3\)
    - Cook, *In pursuit of the Traveling Salesman*\(^4\)
    - Kleinberg and Tardos, *Algorithm Design*\(^5\)
    - Other papers and books by recommendation or discovery
  - Person
    - Regular meetings with Project Advisor Professor James Aspnes\(^6\)
    - Semi-regular meetings with unofficial advisor Professor Daniel Spielman\(^7\)
    - Meet with Applied Math Professor Eric Denardo (interest TBD)
    - Meet with School of Management Professor Edward H. Kaplan (interest TBD)
    - Possible other professors recommended by those listed above as seen fit.

- Obtain a good set of test data / simulations
  - Planets
    - meet with proper individuals in the Astrophysics Department to see if such tables exist already
    - Extensive online search for planet location/distance test data
  - Other
    - Find/Build a simulation of something akin to Helvig’s moving boats.

- Implement potential strategies
  - Brute Force – for purposes of performance standard
  - Dynamic Programming – various possible methods
  - Approximations of best possible solution

- Time and analyze results
- NP-Hard Classification
  - Potential proof or disproof of moving-body TSP classification in NP-Hard
- Write-up of total results [15-25 page] submitted to Prof Aspnes by last day of reading period
- Website displaying results, with possible interactive moving body TSP solver


\(^6\) http://cs-www.cs.yale.edu/homes/aspnes/

\(^7\) http://cs-www.cs.yale.edu/homes/spielman/