1 Papers

1. Definition of Refinements, Forward Simulations, and Backward Simulations

I needed to read sections 1-4 from Forward and Backward Simulations Part I: Untimed Systems in order to better understand what a forward simulation was. This was in the context of Coq proofs. Section 2 has a standard treatment of sequences, relations, and functions. Konig’s Lemma is awesome. This is a rigorous proof for why there exists an infinite simple path in a particular kind of infinite digraph.

Section 3 starts to throw a lot of notation.

An automaton is denoted by $A$. $A$ is made of states and actions, where a step consists of an action moving from one state to another. A fragment consists of a sequences of states and actions, an execution must start with a start state, and a trace is only the actions of an execution.

Preorders are defined over the subset inclusion operator for sets of traces (which can be finite, infinite, or both). Reflexivity/transitivity is obvious for sets.

Moves, denoted by $\beta$, are sequences of actions of execution fragments (not necessarily executions that begin with start states).

There are three kinds of useful automata:

(a) deterministic, which means there is only one start state and for every valid move, there is only one ending state.

(b) finite invisible nondeterminism (fin), which means that there is a finite number of start states, and for every finite sequence of actions for a beginning state, the image set is finite.

(c) forest, which means that there is a unique execution that leads to every state (like a digraph tree).

$\text{after}(A)$ denotes execution/ending state pairs; $\text{past}(A)$ is the inverse of $\text{after}(A)$.

$\text{beh}(A)$ (behavior of automaton) denotes the action set and possible traces.

A trace property is a more general mathematical concept denoting a set and a prefix closure over that set.

A trace property $P$ is limit-closed if an infinite sequence is in $\text{traces}(P)$ if its finite prefixes also are (it is called limit-closed because the sequence of finite prefixes denotes a “limit”).

A useful theorem:

$\text{beh}(A)$ is limit-closed if $A$ is fin, via Konig’s lemma.

The canonical automaton of a trace property, $\text{can}(P)$, is composed of the finite traces of the trace property.

The refinement, forward, and backward simulations are state mappings with particular useful properties. For instance, the refinement is a function that maps states to states and has steps that correspond
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exactly to traces; the forward simulation states that any forward step is mapped properly, as well as ensuring that each starting state is mapped to some starting state in the codomain; and the backward simulation states that any backward step is mapped properly, as well as ensuring that all images of the starting states must also be starting states in the codomain.